

# 3D $N = 6$ Gauged Supergravity: Admissible Gauge Groups, Vacua and RG Flows

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## Abstract

We study  $N = 6$  gauged supergravity in three dimensions with scalar manifolds  $\frac{SU(4,k)}{S(U(4) \times U(k))}$  for  $k = 1, 2, 3, 4$  in great details. We classify some admissible non-compact gauge groups which can be consistently gauged and preserve all supersymmetries. We give the explicit form of the embedding tensors for these gauge groups as well as study their scalar potentials on the full scalar manifold for each value of  $k = 1, 2, 3, 4$  along with the corresponding vacua. Furthermore, the potentials for the compact gauge groups,  $SO(p) \times SO(6-p) \times SU(k) \times U(1)$  for  $p = 3, 4, 5, 6$ , identified previously in the literature are partially studied on a submanifold of the full scalar manifold. This submanifold is invariant under a certain subgroup of the corresponding gauge group. We find a number of supersymmetric AdS vacua in the case of compact gauge groups. We then consider holographic RG flow solutions in the compact gauge groups  $SO(6) \times SU(4) \times U(1)$  and  $SO(4) \times SO(2) \times SU(4) \times U(1)$  for the  $k = 4$  case. The solutions involving one active scalar can be found analytically and describe operator flows driven by a relevant operator of dimension  $\frac{3}{2}$ . For non-compact gauge groups, we find all types of vacua namely AdS, Minkowski and dS, but there is no possibility of RG flows in the AdS/CFT sense.

# 1 Introduction

Gauged supergravity is interesting on its own right and has many applications in string theory. It can give some insight to the study of flux compactifications, see [1] for a review, as well as many applications in the context of the AdS/CFT correspondence [2]. In the present work, we will concentrate on the latter application. This involves classifying the AdS vacua or AdS critical points of the scalar potential which is in turn, dual to some CFT's living on the boundary of the  $AdS$  space. We will also consider RG flows between these CFT's. On the gravity side, the RG flows are described by domain wall solutions connecting two vacua. We focus our attention on gauged supergravity in three dimensions that is useful in the study of  $AdS_3/CFT_2$  correspondence which recently also has some applications in condensed matter systems [3].

The ungauged supergravity theories in three dimensions with all possible numbers of supersymmetries,  $1 \leq N \leq 16$ , have been constructed in [4], and their gauged version has subsequently been studied in [5]. The gauged supergravity is described in the form of Chern-Simons gauged supergravity in which the gauge fields enter the Lagrangian via Chern-Simons term. All the bosonic degrees of freedom are carried by scalar fields which, for  $N > 4$ , are encoded in a symmetric space of the form  $G/H$  [4] where  $G$  is the global symmetry of the theory, and  $H$  is its the maximal compact subgroup. In this sense, the vector fields with Chern-Simons term will not introduce any new degrees of freedom. Unlike in higher dimensional analogues, the number of gauge fields, or equivalently the dimension of the gauge group, is not fixed. Therefore, there are more possibilities for the choices of gauge groups.

It is more convenient, particularly with the formulation of [5] in which the classification of admissible gauge groups can be done in a  $G$ -covariant way, to classify the possible gauge groups in the Chern-Simons form rather than in the more conventional Yang-Mills gauged supergravity. Of course, the latter is related to some higher dimensional theories via certain dimensional reductions. Apart from the case of non-semisimple gaugings as discussed in [6], the Chern-Simons and Yang-Mills gauged supergravities are not equivalent. This fact has a well-known consequence that a Chern-Simons gauged supergravity with other types of gauge groups, compact and non-compact, cannot be obtained from any known higher dimensional framework. Since we will not consider non-semisimple gauge groups in this work, by gauged supergravity, we always mean Chern-Simons gauged supergravity.

We are interested in  $N = 6$  gauged supergravity in three dimensions has been studied in the context of superconformal gaugings in [7]. In this reference, the conformal gaugings of the  $N = 6$  theory give rise to superconformal field theories in three dimensions which can be regarded as theories on the worldvolume of M2-branes and are relevant in the discussion of  $AdS_4/CFT_3$  correspondence. In the present work, we will study the  $N = 6$  theory in the context of  $AdS_3/CFT_2$

correspondence. In this way, we study  $N = 6$  gauged supergravity as a supergravity theory rather than its global supersymmetry limit. The scalar manifold of this theory is of the form  $\frac{SU(4,k)}{S(U(4) \times U(k))}$  where  $k$  is the number of matter multiplets, see [4] for more detail. We will study four cases namely  $k = 1, 2, 3, 4$ .

The gaugings are implemented by mean of the embedding tensor. This tensor has to satisfy a quadratic and linear constraints in order that the gaugings are consistent and compatible with supersymmetry. The embedding tensor for compact gauge groups has been given in [5]. In this work, we will identify some non-compact gauge groups which can be consistently gauged. We then find the corresponding scalar potentials for each gauge group both compact and non-compact and study their critical points as well. Critical points of gauged supergravities in three dimensions with different numbers of supersymmetries have been obtained in many places [8], [9], [10], [11], [12] and [13].

We will also consider RG flow solutions interpolating between two  $AdS_3$  critical points in the case of compact gauge groups. According to the AdS/CFT correspondence, these solutions describe RG flows between two conformal fixed points of a two dimensional dual field theory. Within the framework of three dimensional gauged supergravities, this study has extensively been explored in a number of previous works [8], [9], [10], [14]. The structure of the critical points allows us to find RG flow solutions with only one active scalar that has a non-trivial dependence on the radial coordinate. This gives rise to a simple flow equation that can be solved analytically.

The paper is organized as follows. We review the formulation of three dimensional gauged supergravity in section 2. After a general discussion, we specify to the case of  $N = 6$  theory. In section 3, we study scalar potentials for compact groups as well as their critical points. In section 4, we classify some non-compact gauge groups which can be consistently gauged. We then find their scalar potentials and the corresponding critical points. The RG flow solutions for  $SO(6) \times SU(4) \times U(1)$  and  $SO(4) \times SO(2) \times SU(4) \times U(1)$  gauge groups in the  $k = 4$  case are given in section 5. We also give some detail of the Euler angle parametrization in the appendix including an explicit example. We end the paper with some comments and summary of the main results.

## 2 Three dimensional gauged supergravity

In this section, we give a brief construction of three dimensional gauged supergravity using the formulation given in [5]. We begin with some general features and useful formulae which play an important role in various places of the paper. We finally specify to the  $N = 6$  theory in which scalar fields are encoded in the symmetric space  $\frac{SU(4,k)}{S(U(k) \times U(4))}$ . We refer the reader to [5] for the full detail of the construction.

## 2.1 General construction

It has been shown in [4] that matter coupled supergravity in three dimensions is in the form of a non-linear sigma model coupled to pure supergravity. The target space manifold of scalars, for  $N > 4$ , is a symmetric space of the form  $G/H$  with global and local symmetry groups  $G$  and  $H$ , respectively. The group  $H$  contains the R-symmetry  $SO(N)$  and is of the form  $H = SO(N) \times H'$ . The  $G$  generators  $t^M$  can be decomposed accordingly into  $(X^{IJ}, X^\alpha, Y^A)$  where  $X^{IJ}$  and  $X^\alpha$  generate  $SO(N)$  and  $H'$ , respectively, and  $Y^A$  are non-compact or coset generators. These generators satisfy the  $G$  algebra

$$\begin{aligned} [X^{IJ}, X^{KL}] &= -4\delta^{[I[K} X^{L]J]}, & [X^{IJ}, Y^A] &= -\frac{1}{2}f^{IJ,AB}Y_B, \\ [X^\alpha, X^\beta] &= f^{\alpha\beta}_\gamma X^\gamma, & [X^\alpha, Y^A] &= h^\alpha_B{}^A Y^B, \\ [Y^A, Y^B] &= \frac{1}{4}f_{IJ}^{AB}X^{IJ} + \frac{1}{8}C_{\alpha\beta}h^{\beta AB}X^\alpha \end{aligned} \quad (1)$$

where  $C_{\alpha\beta}$  and  $h^\alpha_B{}^A$  are an  $H'$  invariant tensor and anti-symmetric tensors defined in [4].

In parametrizing the symmetric space  $G/H$ , we introduce a coset representative  $L$  which transforms under  $G$  and  $H$  as  $L \rightarrow gLh$  where  $g \in G$  and  $h \in H$ . Some useful formulae for a coset space are

$$L^{-1}\partial_i L = \frac{1}{2}Q_i^{IJ}X^{IJ} + Q_i^\alpha X^\alpha + e_i^A Y^A, \quad (2)$$

and

$$L^{-1}t^M L = \frac{1}{2}\mathcal{V}^{MIJ}X^{IJ} + \mathcal{V}_\alpha^M X^\alpha + \mathcal{V}_A^M Y^A. \quad (3)$$

In the above formulae,  $Q_i^{IJ}$  and  $Q_i^\alpha$  are composite connections for  $SO(N)$  and  $H'$ , respectively.  $I, J, \dots = 1, \dots, N$  denote the R-symmetry indices, and  $\alpha, \beta, \dots = 1, \dots, \dim H'$  are  $H'$  adjoint indices. The vielbein  $e_i^A$  can be used to construct the metric on the scalar manifold via

$$g_{ij} = e_i^A e_j^B \delta_{AB}, \quad i, j, A, B = 1, \dots, \dim(G/H) \quad (4)$$

and together with its inverse can be used to interchange between curve and flat target space indices. The  $\mathcal{V}$ 's will be used to define the T-tensors which in turn are needed in order to find the scalar potential. We will come back to this later.

Following [5], gaugings are described by introducing an embedding tensor  $\Theta_{\mathcal{MN}}$ . This tensor is symmetric in its indices and gauge invariant. It acts as a projector on the symmetry group  $G$  to the gauge group  $G_0 \subset G$ . The requirement that the gauge generators given by

$$J_{\mathcal{M}} = \Theta_{\mathcal{MN}} t^N \quad (5)$$

form an algebra imposes the so-called quadratic constraint on the embedding tensor

$$\Theta_{\mathcal{PL}} f^{\mathcal{KL}}{}_{(\mathcal{M}} \Theta_{\mathcal{N})\mathcal{K}} = 0. \quad (6)$$

Furthermore, there is a projection constraint which is a consequence of the requirement that a given gauging is consistent with supersymmetry. In general, this constraint is imposed at the level of the T-tensors defined by

$$T_{AB} = \mathcal{V}_A^{\mathcal{M}} \Theta_{\mathcal{MN}} \mathcal{V}_B^{\mathcal{N}}. \quad (7)$$

The projection constraint, acting only on the  $T^{IJKL}$  component, can be written as

$$\mathbb{P}_{\boxplus} T^{IJ,KL} = 0 \quad (8)$$

where  $\boxplus$  denotes the representation  $\boxplus$  of  $SO(N)$ . One important result from [5] is that for symmetric target spaces, the projection constraint (8) can be uplifted to the condition on the embedding tensor

$$\mathbb{P}_{R_0} \Theta_{\mathcal{MN}} = 0. \quad (9)$$

The representation  $R_0$  of  $G$  arises from decomposing the symmetric product of two adjoint representations of  $G$  under  $G$ . Furthermore, the representation  $R_0$ , when branched under  $SO(N)$ , is a unique representation in the above decomposition that contains the  $\boxplus$  representation of  $SO(N)$ .

From the T-tensors, it is straightforward to compute  $A_1$  and  $A_2$  tensors which are relevant in computing the scalar potential via the following formulae

$$\begin{aligned} A_1^{IJ} &= -\frac{4}{N-2} T^{IM, JM} + \frac{2}{(N-1)(N-2)} \delta^{IJ} T^{MN, MN}, \\ A_2^{IJ} &= \frac{2}{N} T^{IJ} + \frac{4}{N(N-2)} f_j^{M(I} T^{J)M} + \frac{2}{N(N-1)(N-2)} \delta^{IJ} f_j^{KL} T_m^{KL}, \\ V &= -\frac{4}{N} g^2 \left( A_1^{IJ} A_1^{IJ} - \frac{1}{2} N g^{ij} A_{2i}^{IJ} A_{2j}^{IJ} \right) \end{aligned} \quad (10)$$

## 2.2 $N = 6$ gauged supergravity in three dimensions

We now in a position to study some gaugings and their associated vacua of  $N = 6$  gauged supergravity. As mentioned earlier, the scalar manifold is the coset space  $\frac{SU(4, k)}{S(U(4) \times U(k))}$  where  $k$  is the number of matter multiplets.

To parametrize the coset representative  $L$ , we first construct the global symmetry group  $G = SU(4, k)$ . In this paper, we study four cases namely  $k = 1, 2, 3, 4$ . We use the standard  $SU(4 + k)$  generators in the form of generalized Gell-Mann matrices. These matrices can be found in many text books for example [15]. We will denote these matrices as  $c_i$ ,  $i = 1, \dots, (4 + k)^2 - 1$ . The non-compact form  $SU(4, k)$  can be constructed from  $SU(4 + k)$  by the Weyl

unitarity trick. This is achieved by introducing a factor of  $i$  to each generator which is identified to be non-compact. The maximally compact subgroup  $H = S(U(4) \times U(k)) \sim SU(4) \times SU(k) \times U(1)$  can be easily identified from the standard construction of the  $SU(4+k)$  generators. All other generators are then the non-compact ones. We have explicitly identified all non-compact generators for the  $k = 4$  case in the appendix. To obtain the non-compact generators for other values of  $k$ , we simply read them off from the 32  $Y$ 's of the  $k = 4$  case. In the  $k = 1$  case, there are eight non-compact generators which are the first eight generators,  $Y_1, \dots, Y_8$ .  $Y_1, \dots, Y_{16}$  and  $Y_1, \dots, Y_{24}$  are non-compact generators of the  $k = 3$  and  $k = 4$  cases, respectively.

Having identified compact and non-compact generators, we can now use the commutation relation  $[X^{IJ}, Y^A]$  in (1) to find the  $f^{IJ}$  tensor whose components are denoted by  $f_{ij}^{IJ}$  or  $f_{AB}^{IJ}$  in the flat basis. With the normalization of  $Y^A$  to be one, we can explicitly write

$$f_{AB}^{IJ} = -2\text{Tr}(Y^B [X^{IJ}, Y^A]). \quad (11)$$

It is now easy to use *Mathematica* to compute all  $f^{IJ}$ .

For the parametrization of  $L$ , it is very useful to use  $SU(n)$  Euler angle parametrization given in [16]. In [16], the parametrization of  $SU(n+1)$  using  $U(n)$  Euler angles has been given. We can apply this parametrization directly to the coset space of the form  $\frac{SU(n,1)}{SU(n) \times U(1)}$ . For the space of the form  $\frac{SU(n,m)}{SU(n) \times U(m)}$  for  $m \neq 1$ , we can apply the general procedure, explained in [17], in parametrizing a Lie group  $G$  using Euler angles of its subgroup  $H$ . In the present case, we parametrize  $SU(n,m)$  using  $SU(n) \times SU(m) \times U(1)$  Euler angles. We will give the explicit form of  $L$  in each case in the following sections. The detail of the parametrization can be found in the appendix.

To find admissible gauge groups which can be gauged consistently with supersymmetry, the corresponding embedding tensors have to satisfy the two constraints (6) and (9). As mentioned above, equation (8) is equivalent to (9) for symmetric target space. Furthermore, it can be shown that the quadratic constraint (6) is equivalent to the relation

$$2A_1^{IK} A_1^{KJ} - NA_2^{IKi} A_{2i}^{JK} = \frac{1}{N} \delta^{IJ} (2A_1^{KL} A_1^{KL} - NA_2^{KLi} A_{2i}^{KL}). \quad (12)$$

This equivalence has been shown explicitly for  $N = 16$  theory in [18].

These conditions are important in searching for admissible gauge groups. For compact gaugings, some gauge groups have been classified in [5]. The associated embedding tensors are given by

$$\Theta = \Theta_{SO(p)} - \Theta_{SO(6-p)} + \alpha \Theta_{SU(k)} - \frac{4\alpha(k-1) + k(p-3)}{4+k} \Theta_{U(1)} \quad (13)$$

where  $\alpha$ , the relative coupling between  $SU(k)$  and  $SO(p) \times SO(6-p)$ , is a free parameter and  $\Theta_{SO(p)} - \Theta_{SO(6-p)}$  is given by

$$\Theta_{IJ,KL} = \theta \delta_{IJ}^{KL} + \delta_{[I[K} \Xi_{L]J]}, \quad (14)$$

$$\Xi_{IJ} = \begin{cases} 2 \left(1 - \frac{p}{N}\right) \delta_{IJ}, & I \leq p \\ -\frac{2p}{N} \delta_{IJ}, & I > p \end{cases}, \quad \theta = \frac{2p - N}{N}. \quad (15)$$

In this paper, we will study gauge groups of non-compact type. After finding the gauge groups, we study their vacua by studying critical points of the resulting scalar potentials. As we have already seen, the potential can be computed from the  $A_1$  and  $A_2$  tensors which can be obtained from various components of the T-tensors. The T-tensors are computed from the embedding tensor and  $\mathcal{V}$ 's from (3).

For conveniences, we repeat the stationarity condition for finding the critical points of the scalar potential given in [5]

$$3A_1^{IK} A_{2j}^{KJ} + N g^{kl} A_{2k}^{IK} A_{3lj}^{KJ} = 0 \quad (16)$$

where  $A_{3lj}^{KL}$  is defined by

$$A_{3ij}^{IJ} = \frac{1}{N^2} \left[ -2D_{(i} D_{j)} A_1^{IJ} + g_{ij} A_1^{IJ} + A_1^{K[I} f_{ij}^{J]K} + 2T_{ij} \delta^{IJ} - 4D_{[i} T_{j]}^{IJ} - 2T_{k[i} f^{IJk}_{j]} \right]. \quad (17)$$

For supersymmetric critical points, the unbroken supersymmetries are encoded in the condition

$$A_1^{IK} A_1^{KJ} \epsilon^J = -\frac{V_0}{4g^2} \epsilon^I = \frac{1}{N} (A_1^{KJ} A_1^{KJ} - \frac{1}{2} N g^{ij} A_{2i}^{KJ} A_{2i}^{KJ}) \epsilon^I. \quad (18)$$

The amount of unbroken supersymmetries correspond to the number of  $\epsilon^I$  that are eigenvectors of  $A_1^{IJ}$ .

### 3 Compact gauge groups and their vacua

In this section, we study some vacua of  $N = 6$  gauged supergravity with compact gauge groups. The gauge groups considered here are  $SO(p) \times SO(6-p) \times SU(k) \times U(1)$  for  $p = 0, 1, 2, 3$ . We consider four cases,  $k = 1, 2, 3, 4$ , separately.

Before going to the discussion of the scalar potentials and their critical points, we give the R-symmetry generators  $X^{IJ}$  in terms of the generalized Gell-Mann matrices  $c_i$ . We emphasize here that our convention is such that  $c_i$  are anti-hermitian. The first fifteen  $c_i$  matrices are generators of  $SU(4) \sim SO(6)$  for

all  $k = 1, 2, 3, 4$  cases. We obtain  $X^{IJ}$  from  $c_1, \dots, c_{15}$  via the following relation

$$\begin{aligned}
X^{12} &= \frac{1}{2}c_3 + \frac{1}{2\sqrt{3}}c_8 - \frac{1}{\sqrt{6}}c_{15}, & X^{13} &= -\frac{1}{2}(c_2 + c_{14}), & X^{23} &= \frac{1}{2}(c_1 - c_{13}), \\
X^{34} &= \frac{1}{2}c_3 - \frac{1}{2\sqrt{3}}c_8 + \frac{1}{\sqrt{6}}c_{15}, & X^{14} &= \frac{1}{2}(c_1 + c_{13}), & X^{35} &= -\frac{1}{2}(c_6 + c_9), \\
X^{56} &= \frac{1}{\sqrt{3}}c_8 + \frac{1}{\sqrt{6}}c_{15}, & X^{36} &= -\frac{1}{2}(c_7 + c_{10}), & X^{24} &= \frac{1}{2}(c_2 - c_{14}), \\
X^{45} &= \frac{1}{2}(c_7 - c_{10}), & X^{46} &= \frac{1}{2}(c_9 - c_6), & X^{15} &= \frac{1}{2}(c_4 - c_{11}), \\
X^{16} &= \frac{1}{2}(c_5 - c_{12}), & X^{25} &= \frac{1}{2}(c_5 + c_{12}), & X^{26} &= -\frac{1}{2}(c_4 + c_{11}).
\end{aligned} \tag{19}$$

It is easy to verify that these generators satisfy the  $[X^{IJ}, X^{KL}]$  commutator given in (1).

### 3.1 The $k = 1$ case

This is the simplest case namely there is only one matter multiplet. The structure of the scalar manifold is very simple and contains only eight scalars. We can study the scalar potential by parametrizing the full scalar manifold with all eight scalars turned on. With Euler angle parametrization given in [16], the coset representative of  $\frac{SU(4,1)}{SU(4) \times U(1)}$  can be written as

$$L = e^{a_1 c_3} e^{a_2 c_2} e^{a_3 c_3} e^{a_4 c_5} e^{a_5 c_8} e^{a_6 c_{10}} e^{a_7 c_{15}} e^{\frac{i}{\sqrt{2}} b_1 c_{17}}. \tag{20}$$

Notice that the factor of  $i$  in the  $c_{17}$  matrix indicates that the generator corresponding to  $c_{17}$  is non-compact. Since  $k = 1$ , there is no additional factor  $SU(k)$  in the gauge groups. As a consequence, there is no free parameter  $\alpha$ . Together with a very simple structure of the scalar manifold whose scalar manifold involves only one non-compact generator, we expect the resulting scalar potential to be very simple. We will see that this is indeed the case.

With the  $L$  given in (20), it turns out that all gauge groups,  $SO(3) \times SO(3)$  and  $SO(p) \times SO(p-6) \times U(1)$  for  $p = 0, 1, 2$ , give rise to the same potential

$$V = -8g^2[5 + 3 \cosh(\sqrt{2}b_1)]. \tag{21}$$

It is clear that there is only one trivial critical point at  $b_1 = 0$  with  $V_0 = V(b_1 = 0) = -64g^2$ . This critical point preserves all gauge symmetries and supersymmetries. The corresponding supersymmetries of the trivial critical point for the gauge groups containing a factor  $SO(p) \times SO(6-p)$  are given by  $(p, 6-p)$ . From now on, we express the number of supersymmetries, preserved by a given critical point, with the notation  $(n_-, n_+)$  where  $n_-$  and  $n_+$  are number of negative and positive eigenvalues of the corresponding  $A_1$  tensor that satisfy the condition (18). This notation is also related to the number of supersymmetries in the dual two dimensional CFT.



### 3.2 The $k = 2$ case

For  $k > 1$ , there is an additional independent coupling of the gauge group  $SU(k)$ . The relative coupling between the gauge group  $SU(k)$  and  $SO(p) \times SO(6-p)$  is denoted by the parameter  $\alpha$  mentioned in the previous section. Since the number of scalar fields in this case is sixteen which is too difficult to study the scalar potential on the full scalar manifold. We will apply the method introduced in [19]. In this approach, we study the potential on a submanifold of  $\frac{SU(4,2)}{SU(4) \times SU(2) \times U(1)}$  coset space. This submanifold is invariant under a symmetry which is a subgroup of the gauge group. Using a consequence of Schur's lemma, it has been shown in [19] that critical points of the potential restricted on this submanifold are critical point of the potential evaluated on the full scalar manifold as well.

We find some non-trivial critical points on the submanifold with  $U(1)_{\text{diag}}$  symmetry. The scalars in this sector are contained in the coset manifold  $\frac{SU(2,2)}{SU(2) \times SU(2) \times U(1)}$ , so there are eight scalars to be parametrized. For  $SO(6) \times SU(2) \times U(1)$ ,  $SO(4) \times SO(2) \times SU(2) \times U(1)$  and  $SO(3) \times SO(3) \times SU(2) \times U(1)$  gauge groups, the residual gauge symmetry  $U(1)_{\text{diag}}$  is generated by  $X_{12} + X_{56}$  where  $X_{IJ}$  are generators of  $SO(6)$  as usual. Using the Euler angle parametrization, we find that the coset representative in this case is given by

$$L = e^{a_1 c_{33}} e^{a_2 c_{34}} e^{a_3 K_3} e^{a_5 M_1} e^{a_6 M_2} e^{a_4 M_3} e^{\frac{i}{\sqrt{2}} b_1 c_{18}} e^{\frac{i}{\sqrt{2}} b_2 c_{31}} \quad (22)$$

where

$$\begin{aligned} K_3 &= \frac{1}{\sqrt{2}} [c_{33}, c_{34}], & M_1 &= -\frac{1}{2\sqrt{2}} [c_{18}, c_{22}], \\ M_2 &= -\frac{1}{2\sqrt{2}} [c_{19}, c_{22}], & M_3 &= \frac{1}{\sqrt{2}} [M_1, M_2]. \end{aligned} \quad (23)$$

For  $SO(5) \times SU(2) \times U(1)$  gauge group, the generator  $X_{56}$  is not a generator of  $SO(5)$ , so we cannot use the coset representative (22). We then choose the generator of the  $U(1)_{\text{diag}}$  to be  $X_{12} + X_{34}$ . The coset representative for this case is given by

$$L = e^{a_1 \kappa} e^{a_2 c_{14}} e^{a_3 \kappa} e^{a_4 c_{33}} e^{a_5 c_{34}} e^{a_6 \lambda} e^{\frac{i}{\sqrt{2}} b_1 c_{20}} e^{\frac{i}{\sqrt{2}} b_2 c_{31}} \quad (24)$$

where

$$\kappa = \frac{1}{\sqrt{2}} [c_{13}, c_{14}], \quad \lambda = \frac{2}{\sqrt{10}} c_{24} - \frac{3}{\sqrt{15}} c_{35}. \quad (25)$$

We now study critical points of each gauge group, separately.

### 3.2.1 $SO(6) \times SU(2) \times U(1)$ gauging

With the coset representative (22), we find the following potential

$$\begin{aligned}
V = & \frac{1}{8}g^2 \left[ -222 + 32(-3 + 2\alpha + \alpha^2) \cosh(\sqrt{2}b_1) - 2(1 + \alpha)^2 \cosh(2\sqrt{2}b_1) \right. \\
& - 48 \cosh[\sqrt{2}(b_1 - b_2)] - 32\alpha \cosh[\sqrt{2}(b_1 - b_2)] - 16\alpha^2 \cosh[\sqrt{2}(b_1 - b_2)] \\
& + \cosh[2\sqrt{2}(b_1 - b_2)] + 2\alpha \cosh[2\sqrt{2}(b_1 - b_2)] + \alpha^2 \cosh[2\sqrt{2}(b_1 - b_2)] \\
& - 96 \cosh(\sqrt{2}b_2) + 64\alpha \cosh(\sqrt{2}b_2) + 32\alpha^2 \cosh(\sqrt{2}b_2) - 2 \cosh(2\sqrt{2}b_2) \\
& - 4\alpha \cosh(2\sqrt{2}b_2) - 2\alpha^2 \cosh(2\sqrt{2}b_2) - 48 \cosh[\sqrt{2}(b_1 + b_2)] \\
& - 32\alpha \cosh[\sqrt{2}(b_1 + b_2)] - 16\alpha^2 \cosh[\sqrt{2}(b_1 + b_2)] + \cosh[2\sqrt{2}(b_1 + b_2)] \\
& \left. + 2\alpha \cosh[2\sqrt{2}(b_1 + b_2)] + \alpha^2 \cosh[2\sqrt{2}(b_1 + b_2)] - 60\alpha - 30\alpha^2 \right]. \quad (26)
\end{aligned}$$

We find three critical points shown in Table 1.

**Table 1:** Critical points of  $SO(6) \times SU(2) \times U(1)$  gauging for the  $k = 2$  case.

	$b$	$V_0$	unbroken SUSY	unbroken gauge symmetry
I	0	$-64g^2$	(6, 0)	$SO(6) \times SU(2) \times U(1)$
II	$\frac{1}{\sqrt{2}} \cosh^{-1} \left( \frac{\alpha-1}{\alpha+1} \right), \alpha < -1$	$-\frac{16g^2(1+2\alpha)^2}{(1+\alpha)^2}$	(4, 0)	$U(1)_{\text{diag}}$
III	$\frac{1}{\sqrt{2}} \cosh^{-1} \left( \frac{\alpha+3}{\alpha+1} \right), \alpha > -1$	$-\frac{16g^2(3+2\alpha)^2}{(1+\alpha)^2}$	(2, 0)	$U(1)_{\text{diag}}$

$V_0$  is the value of the potential at the corresponding critical point or the cosmological constant. At all critical points,  $b_1 = b_2 = b$  where  $b$  is shown in the table.

### 3.2.2 $SO(5) \times SU(2) \times U(1)$ gauging

In this gauge group, the coset representative is given in (24). The potential turns out to be the same as in equation (26). The critical points are of course similar to those given in Table 1 apart from the fact that the amounts of supersymmetries for critical points I, II and III are (5, 1), (4, 1) and (1, 0), respectively. The positions, values of the cosmological constant and unbroken gauge symmetry of these critical points are the same as those in the  $SO(6) \times SU(2) \times U(1)$  gauging.

### 3.2.3 $SO(4) \times SO(2) \times SU(2) \times U(1)$ gauging

The coset representative for this case is given in (22). We find the potential

$$\begin{aligned}
V = & -\frac{1}{8}g^2 \left[ 192 + 30\alpha^2 - 32(-4 + \alpha^2) \cosh(\sqrt{2}b_1) + 2\alpha^2 \cosh(2\sqrt{2}b_1) \right. \\
& + 32 \cosh[\sqrt{2}(b_1 - b_2)] + 16\alpha^2 \cosh[\sqrt{2}(b_1 - b_2)] - \alpha^2 \cosh[2\sqrt{2}(b_1 - b_2)] \\
& + 128 \cosh(\sqrt{2}b_2) - 32\alpha^2 \cosh(\sqrt{2}b_2) + 2\alpha^2 \cosh(2\sqrt{2}b_2) \\
& + 32 \cosh[\sqrt{2}(b_1 + b_2)] + 16\alpha^2 \cosh[\sqrt{2}(b_1 + b_2)] \\
& \left. - \alpha^2 \cosh[2\sqrt{2}(b_1 + b_2)] \right]. \quad (27)
\end{aligned}$$

As in the  $SO(6) \times SU(2) \times U(1)$  gauging, we find three critical points shown in Table 2.

**Table 2:** Critical points of  $SO(4) \times SO(2) \times SU(2) \times U(1)$  gauging for the  $k = 2$  case.

	$b$	$V_0$	unbroken SUSY	unbroken gauge symmetry
I'	0	$-64g^2$	(4, 2)	$SO(4) \times SO(2) \times SU(2) \times U(1)$
II'	$\frac{1}{\sqrt{2}} \cosh^{-1} \left( \frac{\alpha+2}{\alpha} \right), \alpha > 0$	$-\frac{16g^2(1+2\alpha)^2}{\alpha^2}$	(2, 2)	$U(1)_{\text{diag}}$
III'	$\frac{1}{\sqrt{2}} \cosh^{-1} \left( \frac{\alpha-2}{\alpha} \right), \alpha < 0$	$-\frac{16g^2(1-2\alpha)^2}{\alpha^2}$	(2, 0)	$U(1)_{\text{diag}}$

As in Table 1,  $b_1 = b_2 = b$  at the critical points.

### 3.2.4 $SO(3) \times SO(3) \times SU(2) \times U(1)$ gauging

With the same coset representative as in the  $SO(4) \times SO(2) \times SU(2) \times U(1)$  gauging, this gauge group gives rise to the same potential as in (27). The three critical points are characterized by the same parameters as those in Table 2 except for the critical points I', II' and III' having (3, 3), (1, 2) and (2, 1) supersymmetries, respectively.

Notice that for  $SO(6) \times SU(2) \times U(1)$  and  $SO(5) \times SU(2) \times U(1)$  gaugings, there is still one non-trivial critical point when  $\alpha = 0$ , in the limit where  $SU(k)$  decouples. This is not the case for  $SO(3) \times SO(3) \times SU(2) \times U(1)$  and  $SO(4) \times SO(2) \times SU(2) \times U(1)$  gaugings. These two gauge groups do not have any non-trivial critical points in our parametrization when  $\alpha = 0$ . As we will see, the same pattern will appear again in the  $k = 3, 4$  cases. This could be an artifact of our specific parametrization, but after various attempts, this seems to be the only one that gives rise to non-trivial critical points while the complication of the computation is still controllable.

## 3.3 The $k = 3$ case

There are 24 scalars in this case. We choose the scalar submanifold with the same residual symmetry as in the previous case,  $U(1)_{\text{diag}}$ . There are again two coset representatives, one for  $SO(5) \times SU(3) \times U(1)$  gauging and the other one for  $SO(p) \times SO(6-p) \times SU(3) \times U(1)$  for  $p = 3, 4, 6$ . Each coset representative contains 12 scalars parametrizing  $\frac{SU(2,3)}{SU(3) \times SU(2) \times U(1)}$  coset.

The coset representative invariant under  $X_{12} + X_{56}$  is given by

$$L = e^{a_1 \Lambda_3} e^{a_2 c_{34}} e^{a_3 \Lambda_3} e^{a_4 c_{45}} e^{a_5 \Lambda_8} e^{a_6 \Lambda_3} e^{a_7 c_{34}} e^{a_8 \Lambda_3} e^{a_9 M_3} e^{a_{10} M_2} e^{\frac{i}{\sqrt{2}} b_1 c_{18}} e^{\frac{i}{\sqrt{2}} b_1 c_{42}} \quad (28)$$

where

$$\Lambda_3 = \frac{2}{\sqrt{10}} c_{24} - \frac{3}{\sqrt{15}} c_{35}, \quad \Lambda_8 = \frac{2}{\sqrt{30}} c_{24} + \frac{2}{3\sqrt{5}} c_{35} - \frac{7}{3\sqrt{7}} c_{48}, \quad (29)$$

and  $M_2$  and  $M_3$  are given in (23). This  $L$  will be used for the  $SO(p) \times SO(6-p) \times SU(3) \times U(1)$  for  $p = 3, 4, 6$  gaugings.

The coset representative for the  $SO(5) \times SU(3) \times U(1)$  gauging is invariant under  $X_{12} + X_{34}$  and given by

$$L = e^{a_1 \Lambda_3} e^{a_2 c_{34}} e^{a_3 \Lambda_3} e^{a_4 c_{45}} e^{a_5 \Lambda_8} e^{a_6 \Lambda_3} e^{a_7 c_{34}} e^{a_8 \Lambda_3} e^{a_9 k} e^{a_{10} c_{14}} e^{\frac{i}{\sqrt{2}} b_1 c_{20}} e^{\frac{i}{\sqrt{2}} b_1 c_{31}} \quad (30)$$

where  $k$  is given in (25) and  $\Lambda_3$  and  $\Lambda_8$  are given in (29).

The scalar potential and structure of the critical points turn out to be similar to what we have found in the previous case. The only difference lies in the residual gauge symmetry of the critical points. The trivial critical point at  $L = \mathbf{I}$  always preserves the full gauge symmetry. The non-trivial points II, III, II' and III' have  $U(1)_{\text{diag}} \times U(1)$  symmetry for  $SO(3) \times SO(3) \times SU(2) \times U(1)$  and  $SO(5) \times SU(2) \times U(1)$  gaugings and  $U(1)_{\text{diag}} \times U(1) \times U(1)$  symmetry for  $SO(6) \times SU(2) \times U(1)$  and  $SO(4) \times SO(2) \times SU(2) \times U(1)$  gaugings. Other properties are the same as those given in Table 1 and 2.

### 3.4 The $k = 4$ case

In this case, there are 32 scalars which are too difficult to carry out the calculation in the  $U(1)_{\text{diag}}$  invariant sector. We overcome this issue by requiring more residual symmetry namely introducing an  $SU(2) \subset SU(4)$  factor. The scalar manifold we need to parametrize is now invariant under  $U(1)_{\text{diag}} \times SU(2)$ . Notice that in this parametrization, we do not have the limit when  $\alpha \rightarrow 0$  anymore since the residual gauge symmetry of the scalar submanifold contains a subgroup of the  $SU(k) = SU(4)$ . With this extra factor of  $SU(2)$ , we are left with 8 scalars. The coset representatives for the  $SO(p) \times SO(6-p) \times SU(4) \times U(1)$  for  $p = 3, 4, 6$ , and  $SO(5) \times SU(4) \times U(1)$  gaugings are the same as those used in the  $k = 2$  case and given in (22) and (24), respectively.

The scalar potential and structure of the critical points are the same as the  $k = 2$  case except for unbroken gauge symmetries of the non-trivial critical points. Critical points II, III, II' and III' have  $U(1)_{\text{diag}} \times U(1) \times U(1) \times SU(2)$  symmetry for  $SO(6) \times SU(2) \times U(1)$  and  $SO(4) \times SO(2) \times SU(2) \times U(1)$  gaugings and  $U(1)_{\text{diag}} \times U(1) \times SU(2)$  symmetry for  $SO(3) \times SO(3) \times SU(2) \times U(1)$  and  $SO(5) \times SU(2) \times U(1)$  gaugings. Other properties are the same as those given in Table 1 and 2.

## 4 Non-compact gauge groups and their vacua

In this section, we find some non-compact gauge groups by using the consistency conditions given in section 2. For a given gauge group to be admissible, its embedding tensor has to satisfy the conditions (9) and (6), or equivalently, the T-tensors satisfy the conditions (8) and (12). As in the compact case, we study the four cases corresponding to  $k = 1, 2, 3, 4$ , separately. In the results given below, we first give the embedding tensor of each gauge group followed by the

study of the corresponding critical points. Since the scalars corresponding to non-compact generators of the gauge group will drop out from the scalar potential, the number of scalars to be parametrized given below always means the number of scalars described by non-compact generators outside the gauge group. At the trivial critical point where all scalars are zero, the gauge group is broken down to its maximal compact subgroup which constitutes the residual symmetry of the associated critical point. Furthermore, this point preserves all supersymmetries namely  $N = 6$  in three dimensions. As can be seen from (18), supersymmetric critical points are possible only for AdS and Minkowski critical points. It is convenient to express the number of supersymmetries in the case of AdS critical points in the two dimensional language of the corresponding dual CFT's in the form  $(n_-, n_+)$  as in the compact gaugings. In all gauge groups given below, the trivial critical point of AdS type has either  $(6, 0)$  or  $(0, 6)$  supersymmetries while the Minkowski critical point has  $N = 6$  supersymmetry.

#### 4.1 The $k = 1$ case

In this case, the global symmetry group is  $SU(4, 1)$ . We find that the following subgroups can be gauged:

$$SU(3, 1) \times U(1) : \quad \Theta = \Theta_{SU(3,1)} - \frac{3}{5}\Theta_{U(1)} \quad (31)$$

$$SU(2, 1) \times SU(2) \times U(1) : \quad \Theta = \Theta_{SU(2,1)} - \Theta_{SU(2)} - \frac{1}{5}\Theta_{U(1)} \quad (32)$$

$$SU(1, 1) \times SU(3) \times U(1) : \quad \Theta = \Theta_{SU(1,1)} - \Theta_{SU(3)} + \frac{1}{5}\Theta_{U(1)}. \quad (33)$$

Note that for gauge groups which are different real forms of the same complex group, the corresponding embedding tensors are the same. So,  $SU(2, 1) \times SU(2) \times U(1)$  and  $SU(1, 1) \times SU(3) \times U(1)$  have the same embedding tensor. This has been mentioned before in [18] for admissible non-compact gauge groups of the  $N = 16$  theory. We now study scalar potentials for these three gauge groups.

##### 4.1.1 $SU(3, 1) \times U(1)$ gauging

There are six non-compact generators in the gauge group. Scalars corresponding to these generators will drop out from the potential, so we only need to parametrize  $L$  with the remaining two scalars. These correspond to non-compact generators of  $SU(1, 1) \subset SU(3, 1)$ . We then choose the parametrization

$$L = e^{aX} e^{\frac{i}{\sqrt{2}}bc_{16}} e^{-aX}, \quad X = -\frac{1}{\sqrt{2}}[c_{16}, c_{17}]. \quad (34)$$

The potential is given by

$$V = 8g^2(3 \cosh(\sqrt{2}b) - 5). \quad (35)$$

There is no non-trivial critical point. At  $b = 0$ , we find  $V_0 = -16g^2$ .

#### 4.1.2 $SU(2, 1) \times SU(2) \times U(1)$ gauging

There are four relevant scalars described by non-compact generators of  $SU(2, 1) \subset SU(4, 1)$ . We emphasize here that this  $SU(2, 1)$  subgroup is not the same as that appears in the gauge group. The parametrization of  $L$  is

$$L = e^{a_1 q_1} e^{a_2 q_2} e^{a_3 q_3} e^{-\frac{i}{\sqrt{2}} b c_{16}} e^{-a_3 q_3} e^{-a_2 q_2} e^{-a_1 q_1} \quad (36)$$

where

$$q_i = \frac{1}{2} c_i. \quad (37)$$

We find the potential

$$V = 8g^2 \left[ -1 + \cosh(\sqrt{2}b) \right] \quad (38)$$

which does not admit any non-trivial critical points. The trivial critical point is given by  $b = 0$  with  $V_0 = 0$ .

#### 4.1.3 $SU(1, 1) \times SU(3) \times U(1)$ gauging

There are six scalars in  $L$ . They correspond to non-compact generators of  $SU(3, 1) \subset SU(4, 1)$ . With  $U(3)$  Euler angles, we can parametrize  $L$  as

$$L = e^{a_1 c_3} e^{a_2 c_2} e^{a_3 c_3} e^{a_4 c_5} e^{a_5 c_8} e^{\frac{i}{\sqrt{2}} b c_{17}}. \quad (39)$$

We find the scalar potential

$$V = -8g^2 \left( 1 + \cosh(\sqrt{2}b) \right). \quad (40)$$

At  $b = 0$ , there is a trivial critical point with  $V_0 = -16g^2$ .

## 4.2 The $k = 2$ case

In this case, the global symmetry group is  $SU(4, 2)$ . We find that the following gauge groups are admissible:

$$SU(3, 2) \times U(1) : \quad \Theta = \Theta_{SU(3,2)} - \frac{2}{3} \Theta_{U(1)} \quad (41)$$

$$SU(2, 2) \times SU(2) \times U(1) : \quad \Theta = \Theta_{SU(2,2)} - \Theta_{SU(2)} - \frac{1}{3} \Theta_{U(1)} \quad (42)$$

$$SU(1, 2) \times SU(3) : \quad \Theta = \Theta_{SU(1,2)} - \Theta_{SU(3)} \quad (43)$$

$$SU(3, 1) \times SU(1, 1) \times U(1) : \quad \Theta = \Theta_{SU(3,1)} - \Theta_{SU(1,1)} - \frac{1}{3} \Theta_{U(1)} \quad (44)$$

$$SU(2, 1) \times SU(2, 1) : \quad \Theta = \Theta_{SU(2,1)} - \Theta_{SU(2,1)} \quad (45)$$

$$SU(4, 1) \times U(1) : \quad \Theta = \Theta_{SU(4,2)} - \frac{2}{3} \Theta_{U(1)}. \quad (46)$$

We now compute the associated scalar potentials.

#### 4.2.1 $SU(3, 2) \times U(1)$ gauging

There are four relevant scalars parametrizing the manifold  $\frac{SU(2,1)}{SU(2) \times U(1)}$  whose coset representative is given by

$$L = e^{a_1 Q_1} e^{a_2 Q_2} e^{a_3 Q_3} e^{\frac{i}{\sqrt{2}} b c_{16}} \quad (47)$$

where

$$Q_1 = \frac{1}{2} c_{33}, \quad Q_2 = \frac{1}{2} c_{34}, \quad Q_3 = \frac{1}{2} \left( \frac{2}{\sqrt{10}} c_{24} - \frac{3}{\sqrt{15}} c_{35} \right). \quad (48)$$

We obtain the potential

$$V = 8g^2 \left( -5 + 3 \cosh(\sqrt{2}b) \right) \quad (49)$$

The only one critical point is given by  $b = 0$  and  $V_0 = -16g^2$ .

#### 4.2.2 $SU(2, 2) \times SU(2) \times U(1)$ gauging

There are eight scalars which do not correspond to non-compact generators of the gauge group. These scalars form the smaller coset space  $\frac{SU(2,2)}{SU(2) \times SU(2) \times U(1)}$ . The coset representative takes the form

$$L = e^{a_1 P_1} e^{a_2 P_2} e^{a_3 P_3} e^{a_4 Q_1} e^{a_5 Q_2} e^{a_6 Q_3} e^{\frac{i}{\sqrt{2}} b_1 c_{16}} e^{\frac{i}{\sqrt{2}} b_2 c_{27}}, \quad (50)$$

where  $P_i = \frac{1}{2} c_i$  and  $Q_i$ 's are given in (48). We find the potential

$$V = -8g^2 \left[ 3 + \cosh(\sqrt{2}b_1)(-2 + \cosh(\sqrt{2}b_2)) - 2 \cosh(\sqrt{2}b_2) \right] \quad (51)$$

There are two critical points:

- at  $b_1 = b_2 = 0$  with  $V_0 = 0$
- at  $b_1 = b_2 = \frac{1}{\sqrt{2}} \cosh 2$  with  $V_0 = 8g^2$ . This critical point is invariant under  $SU(2) \times U(1)$  symmetry.

#### 4.2.3 $SU(1, 2) \times SU(3)$ gauging

There are twelve relevant scalars in the coset representative which takes the form

$$L = e^{a_1 c_3} e^{a_2 c_2} e^{a_3 c_3} e^{a_4 c_5} e^{\frac{1}{\sqrt{3}} a_5 c_8} e^{a_6 c_3} e^{a_7 c_2} e^{a_8 c_3} e^{a_9 Q_3} e^{a_{10} Q_2} e^{\frac{i}{\sqrt{2}} b_1 c_{16}} e^{\frac{i}{\sqrt{2}} b_2 c_{27}}, \quad (52)$$

where  $Q_i$ 's are given in (48). The scalars correspond to non-compact generators of  $SU(3, 2) \subset SU(4, 2)$ . The potential is found to be

$$V = -8g^2 \left[ 1 + \cosh(\sqrt{2}b_1) \cosh(\sqrt{2}b_2) \right]. \quad (53)$$

There is a trivial critical point at  $b_1 = b_2 = 0$  with  $V_0 = -16g^2$ .

#### 4.2.4 $SU(3, 1) \times SU(1, 1) \times U(1)$ gauging

There are eight relevant scalars encoded in the  $\frac{SU(1,1)}{U(1)} \times \frac{SU(3,1)}{SU(3) \times U(1)}$  coset. The coset representative is parametrized by

$$L = e^{-\frac{1}{2\sqrt{2}}a_1[c_{16}, c_{17}]} e^{\frac{i}{\sqrt{2}}b_1c_{16}} e^{a_2w_3} e^{a_3w_2} e^{a_4w_3} e^{a_5w_5} e^{\frac{1}{\sqrt{3}}a_6w_8} e^{\frac{i}{\sqrt{2}}b_2c_{28}}, \quad (54)$$

where

$$\begin{aligned} w_2 &= \frac{1}{2}c_7, & w_3 &= -\frac{1}{4}(c_3 - \sqrt{3}c_8), \\ w_5 &= \frac{1}{2}c_{12}, & w_8 &= \frac{1}{4}(\sqrt{3}c_3 + c_8 - 4\sqrt{2}c_{15}). \end{aligned} \quad (55)$$

The potential is given by

$$V = 8g^2 \left[ -3 - 2 \cosh(\sqrt{2}b_2) + \cosh(\sqrt{2}b_1)(2 + \cosh(\sqrt{2}b_2)) \right]. \quad (56)$$

There is a critical point at  $b_1 = b_2 = 0$  with  $V_0 = -16g^2$ .

#### 4.2.5 $SU(2, 1) \times SU(2, 1)$ gauging

There are eight scalars parametrized by  $\frac{SU(2,1)}{SU(2) \times U(1)} \times \frac{SU(2,1)}{SU(2) \times U(1)}$  where the two  $SU(2, 1) \subset SU(4, 2)$  are different subgroups from those appearing in the gauge group. The coset representative is given by

$$L = e^{a_1q_1} e^{a_2q_2} e^{a_3q_3} e^{\frac{i}{\sqrt{2}}b_1c_{16}} e^{a_4\tilde{w}_1} e^{a_5\tilde{w}_2} e^{a_6\tilde{w}_3} e^{\frac{i}{\sqrt{2}}b_2c_{29}}, \quad (57)$$

where  $q_i$ 's are given in (37) and

$$\tilde{w}_1 = \frac{1}{2}c_{13}, \quad \tilde{w}_2 = \frac{1}{2}c_{14}, \quad \tilde{w}_3 = \frac{1}{2} \left( -\frac{1}{\sqrt{3}}c_8 + \frac{2}{\sqrt{6}}c_{15} \right). \quad (58)$$

We find the potential

$$\begin{aligned} V &= 2g^2 \left[ \cosh[\sqrt{2}(b_1 + b_2)] - \sinh[\sqrt{2}(b_1 + b_2)](1 + \cosh(2\sqrt{2}b_1) + \cosh(2\sqrt{2}b_2) \right. \\ &\quad - 4 \cosh(\sqrt{2}(b_1 + b_2)) + \cosh(2\sqrt{2}(b_1 + b_2)) + \sinh(2\sqrt{2}b_1) + \sinh(2\sqrt{2}b_2) \\ &\quad \left. - 4 \sinh(\sqrt{2}(b_1 + b_2)) + \sinh(2\sqrt{2}(b_1 + b_2)) \right]. \end{aligned} \quad (59)$$

There is no non-trivial critical point. The trivial one is given by  $b_1 = b_2 = 0$  with  $V_0 = 0$ .



#### 4.2.6 $SU(4, 1) \times U(1)$ gauging

In this case, there are eight scalars parametrized by  $\frac{SU(4,1)}{SU(4) \times U(1)}$  whose coset representative is given by

$$L = e^{a_1 c_3} e^{a_2 c_2} e^{a_3 c_3} e^{a_4 c_5} e^{\frac{1}{\sqrt{3}} a_5 c_8} e^{a_6 c_{10}} e^{\frac{1}{\sqrt{6}} a_7 c_{15}} e^{\frac{i}{\sqrt{2}} b c_{26}}. \quad (60)$$

The potential is obtained to be

$$V = -8g^2 \left( 5 + 3 \cosh(\sqrt{2}b) \right) \quad (61)$$

which does not admit any non-trivial critical points. At  $b = 0$ , we find  $V_0 = -64g^2$ .

### 4.3 The $k = 3$ case

In this case, we find the following gauge groups:

$$SU(3, 3) \times U(1) : \quad \Theta = \Theta_{SU(3,3)} - \frac{5}{7} \Theta_{U(1)} \quad (62)$$

$$SU(2, 3) \times SU(2) \times U(1) : \quad \Theta = \Theta_{SU(2,3)} - \Theta_{SU(2)} - \frac{3}{7} \Theta_{U(1)} \quad (63)$$

$$SU(1, 3) \times SU(3) \times U(1) : \quad \Theta = \Theta_{SU(1,3)} - \Theta_{SU(3)} - \frac{1}{7} \Theta_{U(1)} \quad (64)$$

$$SU(3, 2) \times SU(1, 1) \times U(1) : \quad \Theta = \Theta_{SU(3,2)} - \Theta_{SU(1,1)} - \frac{3}{7} \Theta_{U(1)} \quad (65)$$

$$SU(2, 2) \times SU(2, 1) \times U(1) : \quad \Theta = \Theta_{SU(2,2)} - \Theta_{SU(2,1)} - \frac{1}{7} \Theta_{U(1)} \quad (66)$$

$$SU(1, 2) \times SU(3, 1) \times U(1) : \quad \Theta = \Theta_{SU(1,2)} - \Theta_{SU(3,1)} + \frac{1}{7} \Theta_{U(1)} \quad (67)$$

$$SU(4, 1) \times SU(2) \times U(1) : \quad \Theta = \Theta_{SU(4,1)} - \Theta_{SU(2)} - \frac{3}{7} \Theta_{U(1)} \quad (68)$$

$$SU(4, 2) \times U(1) : \quad \Theta = \Theta_{SU(4,2)} - \frac{5}{7} \Theta_{U(1)}. \quad (69)$$

We now study their critical points.

#### 4.3.1 $SU(3, 3) \times U(1)$ gauging

There are six scalars parametrized by  $\frac{SU(3,1)}{SU(3) \times U(1)}$ . The coset representative takes the form

$$L = e^{a_1 L_3} e^{a_2 L_2} e^{a_3 L_3} e^{a_4 L_5} e^{\frac{1}{\sqrt{3}} a_5 L_8} e^{\frac{i}{\sqrt{2}} b c_{17}}, \quad (70)$$

where

$$\begin{aligned} L_2 &= \frac{1}{2} c_{34}, & L_3 &= \frac{1}{10} \left( \sqrt{10} c_{24} - \sqrt{15} c_{35} \right), \\ L_5 &= \frac{1}{2} c_{45}, & L_8 &= -\frac{1}{40} \left( 2\sqrt{15} c_{24} + 2\sqrt{10} c_{35} - 5\sqrt{14} c_{48} \right). \end{aligned} \quad (71)$$

The potential is given by

$$V = 8g^2(3 \cosh(\sqrt{2}b) - 5) \quad (72)$$

which admits only a trivial critical point at  $b = 0$  with  $V_0 = -16g^2$ .

#### 4.3.2 $SU(2, 3) \times SU(2) \times U(1)$ gauging

The twelve scalars parametrizing the coset  $\frac{SU(2,3)}{SU(2) \times SU(3) \times U(1)}$  whose coset representative is given by

$$L = e^{a_1 L_3} e^{a_2 L_2} e^{a_3 L_3} e^{a_4 L_5} e^{\frac{1}{\sqrt{3}} a_5 L_8} e^{a_6 L_3} e^{a_7 L_2} e^{a_8 L_3} e^{a_9 c_3} e^{a_{10} c_2} e^{\frac{i}{2} b_1 c_{16}} e^{\frac{i}{2} b_2 c_{27}}, \quad (73)$$

where  $L_i$ 's are given in (48). We find the potential

$$V = -8g^2[3 + \cosh(\sqrt{2}b_1)(-2 + \cosh(\sqrt{2}b_2)) - 2 \cosh(\sqrt{2}b_2)]. \quad (74)$$

There are two critical point:

- $b_1 = b_2 = 0$  with  $V_0 = 0$
- $b_1 = b_2 = \frac{1}{\sqrt{2}} \cosh 2$  with  $V_0 = 8g^2$  and residual symmetry  $SU(2) \times U(1) \times U(1)$ .

#### 4.3.3 $SU(1, 3) \times SU(3)$ gauging

The eighteen scalars parametrizing  $\frac{SU(3,3)}{SU(3) \times SU(3) \times U(1)}$  coset are encoded in the coset representative

$$L = e^{a_1 q_3} e^{a_2 q_2} e^{a_3 q_3} e^{a_4 q_5} e^{\frac{1}{\sqrt{3}} a_5 q_8} e^{a_6 q_3} e^{a_7 q_2} e^{a_8 q_3} e^{a_9 L_3} e^{a_{10} L_2} e^{a_{11} L_3} e^{a_{12} L_5} e^{\frac{1}{\sqrt{3}} a_{13} L_8} e^{a_{14} L_3} \times e^{a_{15} L_2} e^{\frac{i}{\sqrt{2}} b_1 c_{16}} e^{\frac{i}{\sqrt{2}} b_2 c_{27}} e^{\frac{i}{\sqrt{2}} b_3 c_{40}}, \quad (75)$$

where  $q_i$ 's and  $L_i$ 's are the same as those given in (37) and (71), respectively. The potential is given by

$$V = -4g^2 \left[ \frac{3}{2} - \frac{1}{2} \cosh(2\sqrt{2}b_1) - \frac{1}{2} \cosh(2\sqrt{2}b_2) + \frac{1}{4} \left( -2 + 2 \cosh(\sqrt{2}b_1) + 2 \cosh(\sqrt{2}b_2) + 2 \cosh(\sqrt{2}b_3) \right)^2 - \frac{1}{2} \cosh(2\sqrt{2}b_3) \right]. \quad (76)$$

The only one critical point is given by  $b_1 = b_2 = b_3 = 0$  with  $V_0 = -16g^2$ .

#### 4.3.4 $SU(3, 2) \times SU(1, 1) \times U(1)$ gauging

The ten relevant scalars are described by the coset representative of the coset  $\frac{SU(2,1)}{SU(2) \times U(1)} \times \frac{SU(3,1)}{SU(3) \times U(1)}$ . We choose the parametrization of  $L$  to be

$$L = e^{a_1 c_{33}} e^{a_2 c_{34}} e^{\frac{1}{\sqrt{2}} a_3 [c_{33}, c_{34}]} e^{\frac{i}{\sqrt{2}} b_1 c_{16}} e^{a_4 w_3} e^{a_5 w_2} e^{a_6 w_3} e^{a_7 w_5} e^{\frac{1}{\sqrt{3}} \frac{a_8}{\sqrt{3}} w_8} e^{\frac{i}{\sqrt{2}} b_2 c_{39}}, \quad (77)$$

where  $w_i$ 's are given in (55). The potential is found to be

$$\begin{aligned} V = & 2g^2 \left[ \cosh(\sqrt{2}(b_1 + b_2)) - \sinh(\sqrt{2}(b_1 + b_2))(1 - 4 \cosh(\sqrt{2}b_1) + \cosh(2\sqrt{2}b_1)) \right. \\ & + 4 \cosh(\sqrt{2}b_2) + \cosh(2\sqrt{2}b_2) - 12 \cosh(\sqrt{2}(b_1 + b_2)) + \cosh(2\sqrt{2}(b_1 + b_2)) \\ & + 4 \cosh(\sqrt{2}(2b_1 + b_2)) - 4 \cosh(\sqrt{2}(b_1 + 2b_2)) - 4 \sinh(\sqrt{2}b_1) + \sinh(2\sqrt{2}b_1) \\ & + 4 \sinh(\sqrt{2}b_2) + \sinh(2\sqrt{2}b_2) - 12 \sinh(\sqrt{2}(b_1 + b_2)) + \sinh(2\sqrt{2}(b_1 + b_2)) \\ & \left. + 4 \sinh(\sqrt{2}(2b_1 + b_2)) - 4 \sinh(\sqrt{2}(b_1 + 2b_2)) \right]. \quad (78) \end{aligned}$$

There is a trivial critical point at  $b_1 = b_2 = 0$  with  $V_0 = -16g^2$ .

#### 4.3.5 $SU(2, 2) \times SU(2, 1) \times U(1)$ gauging

The twelve scalars are parametrized by the following coset representative

$$L = e^{a_1 q_1} e^{a_2 q_2} e^{a_3 q_3} e^{a_4 Q_1} e^{a_5 Q_2} e^{a_6 Q_3} e^{\frac{i}{\sqrt{2}} b_1 c_{16}} e^{\frac{i}{\sqrt{2}} b_2 c_{27}} e^{a_7 Z} e^{a_8 c_{14}} e^{a_9 Z} e^{\frac{i}{\sqrt{2}} b_3 c_{40}}, \quad (79)$$

where  $q_i = \frac{1}{2} c_i$ ,

$$Z = \frac{1}{\sqrt{2}} [c_{13}, c_{14}], \quad (80)$$

and  $Q_i$ 's are given in (48). This  $L$  describes the coset  $\frac{SU(2,2)}{SU(2) \times SU(2) \times U(1)} \times \frac{SU(2,1)}{SU(2) \times U(1)}$ . The potential is given by

$$\begin{aligned} V = & -g^2 \left[ 6 - 2 \cosh(2\sqrt{2}b_1) - 2 \cosh(2\sqrt{2}b_2) - 2 \cosh(2\sqrt{2}b_3) \right. \\ & \left. + 4 \left( -1 + \cosh(\sqrt{2}b_1) + \cosh(\sqrt{2}b_2) - \cosh(\sqrt{2}b_3) \right)^2 \right]. \quad (81) \end{aligned}$$

There two critical points:

- at  $b_1 = b_2 = b_3 = 0$  with  $V_0 = 0$
- at  $b_1 = b_2 = \frac{1}{\sqrt{2}} \cosh 2$ ,  $b_3 = 0$  with  $V_0 = 8g^2$  and residual symmetry  $SU(2) \times U(1) \times U(1)$ .

#### 4.3.6 $SU(1, 2) \times SU(3, 1) \times U(1)$ gauging

The fourteen scalars describing the coset  $\frac{SU(1,1)}{U(1)} \times \frac{SU(3,2)}{SU(3) \times SU(2) \times U(1)}$  are parametrized by

$$L = e^{a_1 c_3} e^{a_2 c_2} e^{a_3 c_3} e^{a_4 c_5} e^{\frac{1}{\sqrt{3}} a_5 c_8} e^{a_6 c_3} e^{a_7 c_2} e^{a_8 c_3} e^{a_9 Q_3} e^{a_{10} Q_2} e^{\frac{i}{\sqrt{2}} b_1 c_{16}} e^{\frac{i}{\sqrt{2}} b_2 c_{27}} \times e^{\frac{1}{\sqrt{2}} a_{11} [c_{23}, c_{24}]} e^{\frac{i}{\sqrt{2}} b_3 c_{42}}, \quad (82)$$

where  $Q_i$ 's are given in (48). The potential is given by

$$V = -g^2 \left[ 6 - 2 \cosh(2\sqrt{2}b_1) - 2 \cosh(2\sqrt{2}b_2) - 2 \cosh(2\sqrt{2}b_3) + 4 \left( 1 + \cosh(\sqrt{2}b_1) + \cosh(\sqrt{2}b_2) - \cosh(\sqrt{2}b_3) \right)^2 \right]. \quad (83)$$

The trivial critical point at  $b_1 = b_2 = b_3 = 0$  has  $V_0 = -16g^2$ .

#### 4.3.7 $SU(4, 1) \times SU(2) \times U(1)$ gauging

The sixteen scalars parametrize the coset  $\frac{SU(4,2)}{SU(4) \times SU(2) \times U(1)}$  whose coset representative takes the form

$$L = e^{a_1 q_3} e^{a_2 q_2} e^{a_3 q_3} e^{a_4 q_5} e^{\frac{1}{\sqrt{3}} a_5 q_8} e^{a_6 q_{10}} e^{a_7 q_3} e^{a_8 q_2} e^{a_9 q_3} e^{a_{10} q_5} e^{\frac{1}{\sqrt{3}} a_{11} q_8} e^{a_{12} \tilde{q}_3} e^{a_{13} \tilde{q}_2} \times e^{a_{14} \tilde{q}_3} e^{\frac{i}{\sqrt{2}} b_1 c_{25}} e^{\frac{i}{\sqrt{2}} b_2 c_{38}} \quad (84)$$

where  $q_i = \frac{1}{2}c_i$  and

$$\tilde{q}_2 = \frac{1}{2}c_{47}, \quad \tilde{q}_3 = -\frac{1}{12} \left( \sqrt{15}c_{35} - \sqrt{21}c_{48} \right). \quad (85)$$

We find the corresponding potential

$$V = -8g^2 \left( 3 + 2 \cosh(\sqrt{2}b_2) + \cosh(\sqrt{2}b_1)(2 + \cosh(\sqrt{2}b_2)) \right). \quad (86)$$

The only one critical point is given by  $b_1 = b_2 = 0$  with  $V_0 = -64g^2$ .

#### 4.3.8 $SU(4, 2) \times U(1)$ gauging

The coset representative for the eight scalars parametrizing the coset  $\frac{SU(4,1)}{SU(4) \times U(1)}$  takes the form

$$L = e^{a_1 q_3} e^{a_2 q_2} e^{a_3 q_3} e^{a_4 q_5} e^{\frac{1}{\sqrt{3}} a_5 q_8} e^{a_6 q_{10}} e^{\frac{1}{\sqrt{6}} a_7 q_{15}} e^{\frac{i}{\sqrt{2}} b c_{37}}, \quad (87)$$

where  $q_i$ 's are given in (37). We find the following potential

$$V = -8g^2 \left( 5 + 3 \cosh(\sqrt{2}b) \right) \quad (88)$$

which does not admit any non-trivial critical points. The trivial critical point is at  $b = 0$  with  $V_0 = -64g^2$ .

## 4.4 The $k = 4$ case

In this case, we find the following gauge groups:

$$SU(3, 4) \times U(1) : \quad \Theta = \Theta_{SU(3,4)} - \frac{3}{4}\Theta_{U(1)} \quad (89)$$

$$SU(2, 4) \times SU(2) \times U(1) : \quad \Theta = \Theta_{SU(2,4)} - \Theta_{SU(2)} - \frac{1}{2}\Theta_{U(1)} \quad (90)$$

$$SU(1, 4) \times SU(3) \times U(1) : \quad \Theta = \Theta_{SU(1,4)} - \Theta_{SU(3)} - \frac{1}{4}\Theta_{U(1)} \quad (91)$$

$$SU(3, 3) \times SU(1, 1) \times U(1) : \quad \Theta = \Theta_{SU(3,3)} - \Theta_{SU(1,1)} - \frac{1}{2}\Theta_{U(1)} \quad (92)$$

$$SU(2, 3) \times SU(2, 1) \times U(1) : \quad \Theta = \Theta_{SU(2,3)} - \Theta_{SU(2,1)} - \frac{1}{4}\Theta_{U(1)} \quad (93)$$

$$SU(1, 3) \times SU(3, 1) : \quad \Theta = \Theta_{SU(1,3)} - \Theta_{SU(3,1)} \quad (94)$$

$$SU(2, 2) \times SU(2, 2) : \quad \Theta = \Theta_{SU(2,2)} - \Theta_{SU(2,2)}. \quad (95)$$

We then move to the study of their critical points.

### 4.4.1 $SU(3, 4) \times U(1)$ gauging

There are eight scalars described by the coset  $\frac{SU(4,1)}{SU(4) \times U(1)}$  whose coset representative is given by

$$L = e^{a_1 j_3} e^{a_2 j_2} e^{a_3 j_3} e^{a_4 j_5} e^{\frac{1}{\sqrt{3}} a_5 j_8} e^{a_6 j_{10}} e^{\frac{1}{\sqrt{6}} a_7 j_{15}} e^{\frac{i}{\sqrt{2}} b c_{17}}, \quad (96)$$

where

$$\begin{aligned} j_2 &= \frac{1}{2} c_{34}, & j_3 &= \frac{1}{2\sqrt{2}} [c_{33}, c_{34}], & j_5 &= \frac{1}{2} c_{45}, \\ j_8 &= \frac{1}{15} (3\sqrt{10} c_{24} + 2\sqrt{15} c_{35} - 5\sqrt{21} c_{48}), & j_{10} &= \frac{1}{2} c_{58}, \\ j_{15} &= \frac{1}{105} (21\sqrt{10} c_{24} + 14\sqrt{15} c_{35} + 10\sqrt{21} c_{48} - 90\sqrt{7} c_{63}). \end{aligned} \quad (97)$$

We find the potential

$$V = 8g^2 \left( -5 + 3 \cosh(\sqrt{2}b) \right) \quad (98)$$

whose only one critical point is characterized by  $b = 0$  and  $V_0 = -16g^2$ .

### 4.4.2 $SU(2, 4) \times SU(2) \times U(1)$ gauging

The sixteen scalars are described by the coset  $\frac{SU(4,2)}{SU(4) \times SU(2) \times U(1)}$ . The coset representative takes the form

$$\begin{aligned} L = & e^{a_1 j_3} e^{a_2 j_2} e^{a_3 j_3} e^{a_4 j_5} e^{\frac{1}{\sqrt{3}} a_5 j_8} e^{a_6 j_3} e^{a_7 j_3} e^{a_8 j_2} e^{a_9 j_3} e^{a_{10} j_5} e^{\frac{1}{\sqrt{3}} a_{11} j_8} e^{a_{12} c_3} e^{a_{13} c_2} \times \\ & e^{a_{14} c_3} e^{\frac{i}{\sqrt{2}} b_1 c_{16}} e^{\frac{i}{\sqrt{2}} b_2 c_{27}}, \end{aligned} \quad (99)$$

where  $j_i$ 's are given in (97). The corresponding potential is given by

$$V = -8g^2 \left[ 3 + \cosh(\sqrt{2}b_1)(-2 + \cosh(\sqrt{2}b_2)) - 2 \cosh(\sqrt{2}b_2) \right]. \quad (100)$$

There are two critical points:

- at  $b_1 = b_2 = 0$  with  $V_0 = 0$
- at  $b_1 = b_2 = \frac{1}{\sqrt{2}} \cosh 2$  with  $V_0 = 8g^2$  and residual symmetry  $SU(2) \times SU(2) \times U(1) \times U(1)$ .

#### 4.4.3 $SU(1,4) \times SU(3) \times U(1)$ gauging

All twenty four scalars are encoded in the coset  $\frac{SU(4,3)}{SU(4) \times SU(3) \times U(1)}$ . The coset representative takes the form

$$L = e^{a_1 c_3} e^{a_2 c_2} e^{a_3 c_3} e^{a_4 c_5} e^{\frac{1}{\sqrt{3}} a_5 c_8} e^{a_6 c_3} e^{a_7 c_3} e^{a_8 j_3} e^{a_9 j_2} e^{a_{10} j_3} e^{a_{11} j_5} e^{\frac{1}{\sqrt{3}} a_{12} j_8} e^{a_{13} j_{10}} e^{a_{14} j_3} \times \\ e^{a_{14} j_3} e^{a_{15} j_2} e^{a_{16} j_3} e^{a_{17} j_5} e^{\frac{1}{\sqrt{3}} a_{18} j_8} e^{a_{19} j_3} e^{a_{20} j_2} e^{a_{21} j_3} e^{\frac{i}{\sqrt{2}} b_1 c_{16}} e^{\frac{i}{\sqrt{2}} b_2 c_{27}} e^{\frac{i}{\sqrt{2}} b_3 c_{40}}, \quad (101)$$

where  $j_i$ 's are given in (97). We find the potential

$$V = -8g^2 \left[ 2 + \cosh(\sqrt{2}b_2)(-1 + \cosh(\sqrt{2}b_3)) - \cosh(\sqrt{2}b_3) + \cosh(\sqrt{2}b_1) \times \right. \\ \left. (-1 + \cosh(\sqrt{2}b_2) + \cosh(\sqrt{2}b_3)) \right]. \quad (102)$$

The only one critical point is at  $b_1 = b_2 = b_3 = 0$  with  $V_0 = -16g^2$ .

#### 4.4.4 $SU(3,3) \times SU(1,1) \times U(1)$ gauging

In this case, there are twelve scalars described by the coset  $\frac{SU(3,1)}{SU(3) \times U(1)} \times \frac{SU(3,1)}{SU(3) \times U(1)}$ . The coset representative takes the form

$$L = e^{a_1 w_3} e^{a_2 w_2} e^{a_3 w_3} e^{a_4 w_5} e^{\frac{1}{\sqrt{3}} a_5 w_8} e^{\frac{i}{\sqrt{2}} b_1 c_{52}} e^{a_6 L_3} e^{a_7 L_2} e^{a_8 L_3} e^{a_9 L_5} e^{\frac{1}{\sqrt{3}} a_{10} L_8} e^{\frac{i}{\sqrt{2}} b_2 c_{17}}, \quad (103)$$

where  $w_i$ 's and  $L_i$ 's are given in (55) and (71), respectively. We find the potential

$$V = 8g^2 [-3 + \cosh(\sqrt{2}b_1)(-2 + \cosh(\sqrt{2}b_2)) + 2 \cosh(\sqrt{2}b_2)] \quad (104)$$

whose trivial critical point is characterized by  $b_1 = b_2 = 0$  with  $V_0 = -16g^2$ .

#### 4.4.5 $SU(2,3) \times SU(2,1) \times U(1)$ gauging

The sixteen scalars described by the coset  $\frac{SU(2,1)}{SU(2) \times U(1)} \times \frac{SU(3,2)}{SU(3) \times SU(2) \times U(1)}$  are parametrized by the coset representative

$$L = e^{a_1 L_3} e^{a_2 L_2} e^{a_3 L_3} e^{a_4 L_5} e^{\frac{1}{\sqrt{3}} a_5 L_8} e^{a_6 L_3} e^{a_7 L_2} e^{a_8 L_3} e^{a_9 q_3} e^{a_{10} q_2} e^{\frac{i}{\sqrt{2}} b_2 c_{16}} e^{\frac{i}{\sqrt{2}} b_3 c_{27}} \times \\ e^{a_{11} z_1} e^{a_{12} z_2} e^{a_{13} z_3} e^{\frac{i}{\sqrt{2}} b_1 c_{44}}, \quad (105)$$

where

$$z_1 = \frac{1}{2}c_{13}, \quad z_2 = \frac{1}{2}c_{14}, \quad z_3 = \frac{1}{2} \left( \frac{-1}{\sqrt{3}}c_8 + \frac{2}{\sqrt{6}}c_{15} \right), \quad (106)$$

and  $q_i$ 's and  $L_i$ 's are given in (37) and (71), respectively. We find the potential

$$V = 8g^2 \left[ -2 + \cosh(\sqrt{2}b_3) + \cosh(\sqrt{2}b_1)(-1 + \cosh(\sqrt{2}b_2) + \cosh(\sqrt{2}b_3)) \right. \\ \left. - 2 \cosh(\sqrt{2}b_2) \sinh^2 \left( \frac{b_3}{\sqrt{2}} \right) \right]. \quad (107)$$

There are two critical points:

- at  $b_1 = b_2 = b_3 = 0$  with  $V_0 = 0$
- at  $b_1 = 0, b_2 = b_3 = \frac{1}{\sqrt{2}} \cosh 2$  with  $V_0 = 8g^2$  and residual symmetry  $SU(2) \times U(1) \times U(1) \times U(1)$ .

#### 4.4.6 $SU(1, 3) \times SU(3, 1)$ gauging

The twenty scalars parametrizing the coset  $\frac{SU(1,1)}{U(1)} \times \frac{SU(3,3)}{SU(3) \times SU(3) \times U(1)}$  are encoded in the following coset representative

$$L = e^{\frac{1}{\sqrt{2}}a_1[c_{31}, c_{32}]} e^{\frac{i}{\sqrt{2}}b_1c_{56}} e^{a_2c_3} e^{a_4c_3} e^{a_5c_5} e^{\frac{1}{\sqrt{3}}a_6c_8} e^{a_7c_3} e^{a_8c_2} e^{a_9c_3} e^{a_{10}L_3} e^{a_{11}L_2} e^{a_{12}L_3} e^{a_{13}L_5} \times \\ e^{\frac{1}{\sqrt{3}}a_{14}L_8} e^{a_{15}L_3} e^{a_{16}L_2} e^{\frac{i}{\sqrt{2}}b_2c_{16}} e^{\frac{i}{\sqrt{2}}b_3c_{27}} e^{\frac{i}{\sqrt{2}}b_4c_{40}}, \quad (108)$$

where  $L_i$ 's are given in (71). We find the potential

$$V = -g^2 \left[ 6 - 2 \cosh(2\sqrt{2}b_2) - 2 \cosh(2\sqrt{2}b_3) + 4 \left( -\cosh(\sqrt{2}b_1) + \cosh(\sqrt{2}b_2) \right. \right. \\ \left. \left. + \cosh(\sqrt{2}b_3) + \cosh(\sqrt{2}b_4) \right)^2 - 2 \cosh(2\sqrt{2}b_4) - 4 \sinh^2(\sqrt{2}b_1) \right] \quad (109)$$

which admits only a trivial critical point at  $b_1 = b_2 = b_3 = 0$  with  $V_0 = -16g^2$ .

#### 4.4.7 $SU(2, 2) \times SU(2, 2)$ gauging

In this case, the sixteen scalars described by the coset  $\frac{SU(2,2)}{SU(2) \times SU(2) \times U(1)} \times \frac{SU(2,2)}{SU(2) \times SU(2) \times U(1)}$  in which the two  $SU(2, 2) \subset SU(4, 4)$  are different from those appearing in the gauge group can be parametrized as

$$L = e^{a_1q_1} e^{a_2q_2} e^{a_3q_3} e^{a_4Q_1} e^{a_5Q_2} e^{a_6Q_3} e^{\frac{i}{\sqrt{2}}b_1c_{16}} e^{\frac{i}{\sqrt{2}}b_2c_{27}} e^{a_7z_1} e^{a_8z_2} e^{a_9z_3} e^{a_{10}\tilde{z}_1} e^{a_{11}\tilde{z}_2} \times \\ e^{\frac{1}{\sqrt{2}}a_{12}[\tilde{z}_1, \tilde{z}_2]} e^{\frac{i}{\sqrt{2}}b_3c_{40}} e^{\frac{i}{\sqrt{2}}b_4c_{55}}, \quad (110)$$

where

$$\tilde{z}_{13} = \frac{1}{2}c_{61}, \quad \tilde{z}_{14} = \frac{1}{2}c_{62}, \quad (111)$$

and  $q_i$ 's,  $z_i$ 's and  $Q_i$ 's are given in (37), (106) and (48), respectively. The potential is given by

$$V = -g^2 \left[ 8 - 2 \cosh(2\sqrt{2}b_1) - 2 \cosh(2\sqrt{2}b_2) - 2 \cosh(2\sqrt{2}b_3) - 2 \cosh(2\sqrt{2}b_4) \right. \\ \left. + 4 \left( \cosh(\sqrt{2}b_1) + \cosh(\sqrt{2}b_2) - \cosh(\sqrt{2}b_3) - \cosh(\sqrt{2}b_4) \right)^2 \right]. \quad (112)$$

There are two critical points:

- at  $b_1 = b_2 = b_3 = b_4 = 0$  with  $V_0 = 0$
- at  $b_1 = b_2 = \frac{1}{\sqrt{2}} \cosh 2$ ,  $b_3 = b_4 = 0$  with  $V_0 = 8g^2$  which is equivalent to the critical point at  $b_3 = b_4 = \frac{1}{\sqrt{2}} \cosh 2$ ,  $b_1 = b_2 = 0$  with the same  $V_0$ . In both cases, the residual symmetry is  $SU(2) \times SU(2) \times U(1)$ .

## 5 RG flow solutions

In this section, we study RG flow solutions interpolating between some AdS vacua given in section 3. We consider only in the  $k = 4$  case with gauge groups  $SO(6) \times SU(4) \times U(1)$  and  $SO(4) \times SO(2) \times SU(4) \times U(1)$ . The two cases have different scalar potentials. According to what we have discussed in section 3, these are the only two independent potential forms. So, in a sense, the study given in this section is enough since the other cases can be studied in a very similar way. According to the AdS/CFT correspondence, AdS critical points are identified with two dimensional CFT's at the boundary of  $AdS_3$  or conformal fixed points of the dual two dimensional field theory. In an RG flow, a UV CFT is perturbed by turning on some operators, or the operators acquire vev's which break conformal symmetry. If there exist another conformal fixed point in the IR, the flow will drive the UV CFT to another CFT in the IR. The central charge of the corresponding CFT can be found by a well-known result

$$c = \frac{3L}{2G_N} \sim \frac{1}{\sqrt{-V_0}} \quad (113)$$

where we have used the fact that in our case, the  $AdS_3$  radius  $L = \frac{1}{\sqrt{-V_0}}$ .

Because the two non-trivial critical points are supersymmetric, the flow solution can be found by solving BPS equations coming from setting the supersymmetry variation of  $\psi_\mu^I$  and  $\chi^{iI}$  to zero. This solution will describe a supersymmetric RG flow in the dual field theory.

Supersymmetry transformations of  $\psi_\mu^I$  and  $\chi^{iI}$  are given by [5]

$$\delta \psi_\mu^I = \mathcal{D}_\mu \epsilon^I + g A_1^{IJ} \gamma_\mu \epsilon^J, \\ \delta \chi^{iI} = \frac{1}{2} (\delta^{IJ} \mathbf{1} - f^{IJ})^i_j \mathcal{D} \phi^j \epsilon^J - g N A_2^{JI} \epsilon^J \quad (114)$$



where  $D_\mu \epsilon^I = \left( \partial_\mu + \frac{1}{2} \omega_\mu^a \gamma_a + \dots \right) \epsilon^I$ .

As in usual studies of holographic RG flows, the metric ansatz is taken to be

$$ds^2 = e^{2A(r)} dx_{1,1}^2 + dr^2. \quad (115)$$

Recall the result of section 3, we find that the RG flow solution will interpolate between two critical points with  $(b_1, b_2) = (0, 0)$  and  $(b_1, b_2) = (b, b)$  in which the two scalar fields  $b_1$  and  $b_2$  take the same value at both critical points. So, we use the ansatz for the coset representative of the form

$$L = e^{b(r)Y_3} e^{b(r)Y_{15}} \quad (116)$$

where all the  $Y$ 's generators are given in the appendix. It is now straightforward to use this information to compute the BPS equations  $\delta\psi_\mu^I = 0$  and  $\delta\chi^{iI} = 0$ . In writing BPS equations below, we need to impose a projection condition  $\gamma_r \epsilon^I = \epsilon^I$  so that the flow solution preserves half of the supersymmetries.

## 5.1 RG Flows in $SO(6) \times SU(4) \times U(1)$ gauging

In this case, the flow interpolates between  $(6, 0)$  critical point and  $(4, 0)$  or  $(2, 0)$  points depending on the value of  $\alpha$ ,  $\alpha < -1$  or  $\alpha > -1$ , respectively. The UV point is identified with the  $(6, 0)$  point with  $SO(6) \times SU(4) \times U(1)$  symmetry while the IR point is identify with  $(4, 0)$  or  $(2, 0)$  points with  $SU(2) \times U(1) \times U(1) \times U(1)_{\text{diag}}$  symmetry.

### 5.1.1 An RG flow between $(6, 0)$ and $(4, 0)$ critical points

The flow equation coming from  $\delta\chi^{iI} = 0$  is given by

$$\frac{db(r)}{dr} = \sqrt{2}g \left[ 1 - \alpha + (1 + \alpha) \cosh(\sqrt{2}b(r)) \right] \sinh(\sqrt{2}b(r)). \quad (117)$$

We also recall that in this flow,  $\alpha < -1$ . This equation can be solved for  $r$  as a function of  $b$ . The solution is given by

$$\begin{aligned} r = & \frac{1}{4g\alpha} \ln \left[ \cosh \frac{b}{\sqrt{2}} \right] - \frac{1+\alpha}{8g\alpha} \ln \left[ 1 - \alpha + (1 + \alpha) \cosh(\sqrt{2}b) \right] \\ & + \frac{1}{4g} \ln \left[ \sinh \frac{b}{\sqrt{2}} \right]. \end{aligned} \quad (118)$$

We have neglected the additive constant in the  $r$  solution because this can be removed by shifting the coordinate  $r$ .

As  $b = 0$ , we find

$$b(r) \sim e^{4gr} \sim e^{-\frac{r}{2L_{\text{UV}}}}, \quad L_{\text{UV}} = \frac{1}{8|g|}. \quad (119)$$

To identify this with the UV point in which  $r \rightarrow \infty$ , we have chosen  $g < 0$ . This asymptotic behavior of the scalar gives information about the dimension of the operator that drives the flow. The discussion on this point can be found in many references, see for example [20]. In this case, we find that the flow is driven by a relevant operator of dimension  $\Delta = \frac{3}{2}$ .

At the IR point  $b = \frac{1}{\sqrt{2}} \cosh^{-1} \frac{\alpha-1}{\alpha+1}$ , we find

$$b(r) \sim e^{-\frac{8g\alpha r}{1+\alpha}} = e^{\frac{2\alpha}{(1+2\alpha)} \frac{r}{L_{\text{IR}}}}, \quad L_{\text{IR}} = \frac{1+\alpha}{4|g|(1+2\alpha)}. \quad (120)$$

In the IR, the operator becomes irrelevant and has dimension  $\Delta = \frac{2(1+3\alpha)}{1+2\alpha}$  which is greater than two for  $\alpha < -1$ . The ratio of the central charges is given by

$$\frac{c_{\text{UV}}}{c_{\text{IR}}} = \frac{1+2\alpha}{2(1+\alpha)} > 1, \quad \text{for } \alpha < -1. \quad (121)$$

The next thing is to determine the function  $A(r)$  in the metric. This can be done by using  $\delta\psi_\mu^I = 0$ ,  $\mu = 0, 1$ , equation given by

$$\begin{aligned} \frac{dA(r)}{dr} = & -g \left[ 5 + \alpha - (\alpha - 1) \cosh(\sqrt{2}b(r)) \right. \\ & \left. + \cosh(\sqrt{2}b(r)) \left\{ 1 - \alpha + (1 + \alpha) \cosh(\sqrt{2}b(r)) \right\} \right]. \end{aligned} \quad (122)$$

Using  $b(r)$  as an independent variable, we can write the above equation as

$$\frac{dA(b)}{db} = -\frac{5 + \alpha - (\alpha - 1) \cosh(\sqrt{2}b) + \cosh(\sqrt{2}b)(1 - \alpha + (1 + \alpha) \cosh(\sqrt{2}b))}{\sqrt{2} [1 - \alpha + (1 + \alpha) \cosh(\sqrt{2}b)] \sinh(\sqrt{2}b)}. \quad (123)$$

The solution is easily obtained to be

$$\begin{aligned} A = & \frac{1+2\alpha}{2\alpha} \ln \left[ 1 - \alpha + (1 + \alpha) \cosh(\sqrt{2}b) \right] - 2 \ln \left[ 2 \sinh \frac{b}{\sqrt{2}} \right] \\ & - \frac{1+\alpha}{\alpha} \ln \left[ 2 \cosh \frac{b}{\sqrt{2}} \right]. \end{aligned} \quad (124)$$

We have again neglected the additive constant by rescaling the coordinates  $x^\mu$  for  $\mu = 0, 1$ .

We can readily check from the kinetic term that our scalar  $b(r)$  is canonically normalized, so we can read off the value of mass squared from the scalar potential. To quadratic order near the UV point, the potential (26) is given by

$$V = -64g^2 - 48g^2b^2 \quad (125)$$

which gives  $m^2 L_{\text{UV}}^2 = -\frac{3}{4}$ . We find that the relation  $m^2 L^2 = \Delta(\Delta - 2)$  precisely gives  $\Delta = \frac{3}{2}$  consistent with what we have found before from the asymptotic

behavior of the scalar  $b(r)$ .

At the IR point, we obtain the potential to second order

$$V = -\frac{16g^2(1+2\alpha)^2}{(1+\alpha)^2} + \frac{64g^2\alpha(1+3\alpha)}{(1+\alpha)^2}b^2. \quad (126)$$

We find  $m^2 L_{\text{IR}}^2 = \frac{4\alpha(1+3\alpha)}{(1+2\alpha)^2}$  or  $\Delta = \frac{2(1+3\alpha)}{1+2\alpha}$ .

### 5.1.2 An RG flow between $(6, 0)$ and $(2, 0)$ critical points

We now consider an RG flow between  $(6, 0)$  and  $(2, 0)$  critical points similar to what we have done in the previous case. In this case,  $\alpha > -1$ .

We begin with the flow equation obtained from  $\delta\chi^{iI} = 0$

$$\frac{db(r)}{dr} = -\sqrt{2}g \left[ -3 - \alpha + (1 + \alpha) \cosh(\sqrt{2}b(r)) \right] \sinh(\sqrt{2}b(r)). \quad (127)$$

As before, we can solve for  $r$  as a function of  $b$  and find

$$\begin{aligned} r = & -\frac{1+\alpha}{8g(2+\alpha)} \ln \left[ (1+\alpha) \cosh(\sqrt{2}b) - 3 - \alpha \right] + \frac{1}{4g} \ln \left[ \sinh \frac{b}{\sqrt{2}} \right] \\ & - \frac{1}{4g(2+\alpha)} \ln \left[ \cosh \frac{b}{\sqrt{2}} \right]. \end{aligned} \quad (128)$$

Near the UV point  $b = 0$ , we find

$$b(r) \sim e^{4gr} = e^{-\frac{r}{2L_{\text{UV}}}}, \quad L_{\text{UV}} = \frac{1}{8|g|}. \quad (129)$$

We again choose  $g < 0$  to identify the UV point with  $r \rightarrow \infty$ . The flow is driven by a relevant operator of dimension  $\Delta = \frac{3}{2}$ .

Near the IR point  $b = \frac{1}{\sqrt{2}} \cosh^{-1} \frac{\alpha+3}{\alpha+1}$ , the scalar behaves as

$$b(r) \sim e^{-\frac{8g(2+\alpha)r}{1+\alpha}} = e^{\frac{2(2+\alpha)r}{(3+2\alpha)L_{\text{IR}}}}, \quad L_{\text{IR}} = \frac{1+\alpha}{4|g|(3+2\alpha)}. \quad (130)$$

The operator has dimension  $\Delta = \frac{2(5+3\alpha)}{3+2\alpha} > 2$ , for  $\alpha > -1$ , in the IR.

The equation for  $A(r)$  obtained from  $\delta\psi_\mu^I = 0$  is given by

$$\begin{aligned} \frac{dA(r)}{dr} = & g \left[ \alpha - 3 - (3 + \alpha) \cosh(\sqrt{2}b(r)) \right. \\ & \left. + \cosh(\sqrt{2}b(r)) \{ -3 - \alpha + (1 + \alpha) \cosh(\sqrt{2}b(r)) \} \right] \end{aligned} \quad (131)$$

or, in term of variable  $b$ ,

$$\frac{dA}{db} = -\frac{\alpha - 3 - (3 + \alpha) \cosh(\sqrt{2}b) + \cosh(\sqrt{2}b) [-\alpha - 3 + (1 + \alpha) \cosh(\sqrt{2}b)]}{\sqrt{2} [-3 - \alpha + (1 + \alpha) \cosh(\sqrt{2}b)] \sinh(\sqrt{2}b)}. \quad (132)$$

The solution for  $A(b)$  is found to be

$$A = -\frac{1+\alpha}{2+\alpha} \ln \left[ 2 \cosh \frac{b}{\sqrt{2}} \right] + \frac{3+2\alpha}{2(2+\alpha)} \ln \left[ -3 - \alpha + (1+\alpha) \cosh(\sqrt{2}b) \right] - 2 \ln \left[ 2 \sinh \frac{b}{\sqrt{2}} \right]. \quad (133)$$

The ratio of the central charges is given by

$$\frac{c_{\text{UV}}}{c_{\text{IR}}} = \frac{3+2\alpha}{2(1+\alpha)} > 1, \quad \text{for} \quad \alpha > -1. \quad (134)$$

We can compute the value of mass squared at both end points and find that the dimension of the operator found previously agrees with the result obtained from the relation  $m^2 L^2 = \Delta(\Delta - 2)$ . We will not give a repetition here.

## 5.2 RG Flows in $SO(4) \times SO(2) \times SU(4) \times U(1)$ gauging

In this case, there are three critical points involving in the flows. The trivial critical point at  $b_1 = b_2 = 0$  has  $(4, 2)$  supersymmetry with  $SO(4) \times SO(2) \times SU(4) \times U(1)$  gauge symmetry. The two non-trivial critical points are given by  $b_1 = b_2 = \frac{1}{\sqrt{2}} \cosh^{-1} \frac{2+\alpha}{\alpha}$  with  $(2, 2)$  supersymmetry and  $b_1 = b_2 = \frac{1}{\sqrt{2}} \cosh^{-1} \frac{\alpha-2}{\alpha}$  with  $(2, 0)$  supersymmetry. Both of them have unbroken gauge symmetry  $U(1) \times U(1) \times U(1)_{\text{diag}} \times SU(2)$ .

All procedures involved are the same as in the previous subsection, so we will not go into much detail but simply give key results in each step. Similar to the previous gauge group, there are two possible flows namely the flow between  $(4, 2)$  and  $(2, 2)$  points and between  $(4, 2)$  and  $(2, 0)$  points.

### 5.2.1 An RG flow between $(4, 2)$ and $(2, 2)$ critical points

In this case, the flow interpolates between two critical points with chiral and non-chiral supersymmetries. The  $(2, 2)$  critical point requires  $\alpha > 0$ . The flow equation from  $\delta\chi^{iI} = 0$  reads

$$\frac{db(r)}{dr} = -\sqrt{2}g \left[ 2 + \alpha - \alpha \cosh(\sqrt{2}b(r)) \right] \sinh(\sqrt{2}b(r)). \quad (135)$$

The solution for  $r$  in term of  $b$  is found to be

$$r = \frac{1}{4g(1+\alpha)} \ln \left[ \cosh \frac{b}{\sqrt{2}} \right] + \frac{\alpha}{8g(1+\alpha)} \ln \left[ \alpha \cosh(\sqrt{2}b) - \alpha - 2 \right] - \frac{1}{4g} \ln \left[ \sinh \frac{b}{\sqrt{2}} \right]. \quad (136)$$

Choosing  $g > 0$ , the UV point is described by  $r \rightarrow \infty$ . Near this point, the scalar behaves

$$b(r) \sim e^{-4gr} = e^{-\frac{r}{2L_{\text{UV}}}}, \quad L_{\text{UV}} = \frac{1}{8g}. \quad (137)$$

The flow is seen to be driven by a relevant operator of dimension  $\frac{3}{2}$ .

At the IR point,  $r \rightarrow -\infty$ , we find

$$b(r) \sim e^{\frac{8g(1+\alpha)r}{\alpha}} = e^{\frac{2(1+\alpha)r}{(1+2\alpha)L_{\text{IR}}}}, \quad L_{\text{IR}} = \frac{\alpha}{4g(1+2\alpha)}. \quad (138)$$

The dimension of the operator in the IR is  $\Delta = \frac{2(2+3\alpha)}{1+2\alpha}$  which is greater than two for  $\alpha > 0$ .

We then move to the  $\delta\psi_\mu^I = 0$  equation which gives

$$\frac{dA}{dr} = -\frac{1}{2}g \left[ 8 - 3\alpha + 4(2 + \alpha) \cosh(\sqrt{2}b(r)) - \alpha \cosh(2\sqrt{2}b(r)) \right] \quad (139)$$

or in term of variable  $b$

$$\frac{dA(b)}{db} = -\frac{8 - 3\alpha + 4(2 + \alpha) \cosh(\sqrt{2}b(r)) - \alpha \cosh(2\sqrt{2}b(r))}{2\sqrt{2} [2 + \alpha - \alpha \cosh(\sqrt{2}b)] \sinh(\sqrt{2}b)}. \quad (140)$$

The solution is given by

$$\begin{aligned} A = & \frac{\alpha}{1+\alpha} \ln \left( 2 \cosh \frac{b}{\sqrt{2}} \right) - \frac{1+2\alpha}{2(1+\alpha)} \ln \left[ \alpha \cosh(\sqrt{2}b) - \alpha - 2 \right] \\ & + 2 \ln \left( 2 \sinh \frac{b}{\sqrt{2}} \right). \end{aligned} \quad (141)$$

It is easily seen that as  $b \rightarrow 0$ ,  $A \rightarrow \infty$  and as  $b \rightarrow \frac{1}{\sqrt{2}} \cosh \frac{\alpha+2}{\alpha}$ ,  $A \rightarrow -\infty$ . The ratio of the central charges is given by

$$\frac{c_{\text{UV}}}{c_{\text{IR}}} = \frac{1+2\alpha}{2\alpha} > 1, \quad \text{for } \alpha > 0. \quad (142)$$

### 5.2.2 An RG flow between $(4, 2)$ and $(2, 0)$ critical points

We now give our last RG flow solution. The flow equation reads

$$\frac{db(r)}{dr} = -\sqrt{2}g \left[ \alpha - 2 - \alpha \cosh(\sqrt{2}b(r)) \right] \sinh(\sqrt{2}b(r)) \quad (143)$$

whose solution for  $r(b)$  is given by

$$\begin{aligned} r = & \frac{1}{4g(\alpha-1)} \ln \left( \cosh \frac{b}{\sqrt{2}} \right) - \frac{\alpha}{8g(\alpha-1)} \ln \left[ 2 - \alpha + \alpha \cosh(\sqrt{2}b) \right] \\ & + \frac{1}{4g} \ln \left( \sinh \frac{b}{\sqrt{2}} \right). \end{aligned} \quad (144)$$

Near  $b = 0$ , the UV point, we find

$$b(r) \sim e^{4gr} = e^{-\frac{r}{2L_{\text{UV}}}}, \quad L_{\text{UV}} = \frac{1}{8|g|} \quad (145)$$

where we have chosen  $g < 0$  to identify this as the UV point when  $r \rightarrow \infty$ .

Near the IR point,  $b = \frac{1}{\sqrt{2}} \cosh \frac{\alpha-2}{\alpha}$  and  $r \rightarrow -\infty$ , we find

$$b(r) \sim e^{-\frac{8g(\alpha-1)r}{\alpha}} = e^{\frac{(2\alpha-1)r}{(2\alpha-1)L_{\text{IR}}}}, \quad L_{\text{IR}} = \frac{\alpha}{4|g|(2\alpha-1)}. \quad (146)$$

So, the flow is again driven by a relevant operator of dimension  $\frac{3}{2}$ , and the operator becomes irrelevant in the IR with dimension  $\Delta = \frac{2(2-3\alpha)}{1-2\alpha} > 2$  for  $\alpha < 0$ .

We finally determine the  $A(r)$  function in this case. It is the solution of the  $\delta\psi_\mu^I = 0$  equation given by

$$\frac{dA(r)}{dr} = -\frac{1}{2}g \left[ 8 + 3\alpha - 4(\alpha-2) \cosh(\sqrt{2}b(r)) + \alpha(2\sqrt{2}b(r)) \right] \quad (147)$$

or

$$\frac{dA(b)}{db} = \frac{8 + 3\alpha - 4(\alpha-2) \cosh(\sqrt{2}b(r)) + \alpha(2\sqrt{2}b(r))}{2\sqrt{2} [\alpha - 2 - \alpha \cosh(\sqrt{2}b)] \sinh(\sqrt{2}b)}. \quad (148)$$

The solution is found to be

$$A = \frac{\alpha}{\alpha-1} \ln \left[ 2 \cosh \frac{b}{\sqrt{2}} \right] + \frac{1-2\alpha}{2(\alpha-1)} \ln \left[ 2 - \alpha + \alpha \cosh(\sqrt{2}b) \right] - 2 \ln \left( 2 \sinh \frac{b}{\sqrt{2}} \right). \quad (149)$$

The ratio of the central charges is given by

$$\frac{c_{\text{UV}}}{c_{\text{IR}}} = \frac{2\alpha-1}{2\alpha} > 1, \quad \text{for} \quad \alpha < 0. \quad (150)$$

The analysis of the mass squared from the expansion of the scalar potential near the critical points can be done in the same way as in the previous subsection. We will again not repeat this computation here.

## 6 Conclusions

In this paper, we have extensively studied  $N = 6$  gauged supergravity in three dimensions. We have used the Euler angle parametrization in parametrizing the scalar coset manifold  $\frac{SU(4,k)}{S(U(4) \times U(k))}$  and submanifolds thereof. This parametrization turns out to be very useful. We have identified admissible gauge groups of non-compact type, studied their scalar potentials and found some of the critical

points associated to each scalar potential.

In compact gauge groups identified in [5], we have found a number of supersymmetric AdS vacua. It is possible in this case to discuss holographic RG flow solutions in the context of the AdS/CFT correspondence. We have studied four analytic RG flow solutions in two gauge groups,  $SO(6) \times SU(4) \times U(1)$  and  $SO(4) \times SO(2) \times SU(4) \times U(1)$ , in the  $k = 4$  case. Given the structure of the critical points including the form of the potential, this study is, in a sense, sufficient because among  $k = 2, 3, 4$  cases, there are actually only two different potential forms. The flows in other gauge groups or in different values of  $k$  can be obtained similarly. The resulting solutions involve one active scalar and can be solved analytically. The flows are operator flows driven by a relevant operator of dimension  $\frac{3}{2}$  and respect the c-theorem. Notice that the solutions look very much like the solutions found in [8] and [10].

In non-compact gauge groups, it is remarkable that apart from  $L = \mathbf{I}$ , we have not found any other (non-trivial) AdS critical points whether supersymmetric or not. We strongly believe that there is no non-trivial AdS critical point in the non-compact groups studied here. Therefore, we have no possibilities of RG flows in the AdS/CFT sense. However, it is interesting to study domain wall solutions connecting between dS vacua or even between dS and Minkowski vacua as discussed in [21]. A domain wall between dS vacua could also be interpreted as an RG flow in the context of the dS/CFT correspondence [22]. An example of this study can be found in [23].

Recently, the new approach for finding critical points of gauged supergravities has been introduced in [24]. This technique is based on the variation of the embedding tensor rather than the extremization of the scalar potential as done in this and many other works. New critical points of  $N = 8$  gauged supergravity in four dimensions have been found within this framework. It is naturally interesting to investigate critical points of the  $N = 6$  theory studied here particularly for compact gauge groups, which have not been explored in full details due to the complication of the computation, with this new approach as well as to reexamine the scalar potentials of the theories with different values of  $N$  studied in [8], [9], [10], [11] and [13]. We hopefully expect to find some new vacua of these theories, too. We leave these issues for future works.

### Acknowledgement

This work is partially supported by Faculty of Science, Chulalongkorn University through the A1B1 project and Thailand Center of Excellence in Physics through the ThEP/CU/2-RE3/11 project. The second author would like to thank Chulalongkorn University for the support via Ratchadapisek Sompote Endowment Fund in carrying out this research project.

## A On the Euler angle parametrization

In this appendix, we review the Euler angle parametrization of a Lie group  $G$  using Euler angles of its subgroup  $H$ . We begin with some general idea of this parametrization based on the result of [17] and then specify to our case for the coset of the form  $\frac{SU(n,m)}{SU(n) \times SU(m) \times U(1)}$ . The same procedure can be applied to all cases discussed in this paper. So, we will give only one example namely the coset  $\frac{SU(4,3)}{SU(4) \times SU(3) \times U(1)}$  in the case of  $k = 4$ .

For a Lie group  $G$ , the  $H$  Euler angle parametrization of  $G$  is given by [17]

$$G = Be^V H. \quad (151)$$

$H$  is the parametrization of the subgroup  $H \subset G$  which can in turn be parametrized by Euler angles of some subgroup  $H_1 \subset H$ . However, we will explain only the parametrization of  $G$  itself since our main aim here is to demonstrate the parametrization procedure. In our application with non-compact  $G$ , we would like to parametrize the coset space  $G/H$  which can be obtained from the above parametrization by a quotient with  $H$ . Therefore, in the following, we will choose  $H$  to be the maximal compact subgroup of  $G$ . Following [17], we denote the algebras of  $G$  and  $H$  by  $\mathfrak{g}$  and  $\mathfrak{h}$ , respectively. The  $G$  generators are then decomposed into  $H$  generators and non-compact generators which constitute a subspace  $\mathfrak{p} \subset \mathfrak{g}$ . The subspace  $V \subset \mathfrak{p}$  consists of the minimal set of non-compact generators such that the whole subspace  $\mathfrak{p}$  can be generated by the adjoint action of  $H$  on  $V$ .

The factor  $B = H/H_0$  where  $H_0$  is the redundancy of the parametrization. Obviously, we find  $\dim G = \dim H/H_0 + \dim V + \dim H$ .  $H_0$  consists of the automorphisms of  $V$ . They are not needed to generate the whole  $\mathfrak{p}$  from  $V$ , or equivalently, they commute with  $V$ . We will see how this procedure works in the following example.

We recall that the  $SU(8)$  generators can be constructed from the generalized Gell-Mann matrices  $c_i$ ,  $i = 1, \dots, 63$ . Its non-compact form  $SU(4,4)$  is obtained by multiplying each non-compact generator by a factor of  $i$ . Explicitly,



the non-compact generators for the  $k = 4$  case are given by

$$Y_A = \begin{cases} \frac{1}{\sqrt{2}}c_{A+15}, & A = 1, \dots, 8 \\ \frac{1}{\sqrt{2}}c_{A+16}, & A = 9, \dots, 16 \\ \frac{1}{\sqrt{2}}c_{A+19}, & A = 17, \dots, 24 \\ \frac{1}{\sqrt{2}}c_{A+24}, & A = 25, \dots, 32 \end{cases} \quad (152)$$

We are now in a position to parametrize the coset  $K = \frac{SU(4,3)}{SU(4) \times SU(3) \times U(1)}$  in the case of  $SU(1,4) \times SU(3) \times U(1)$  gauging. The subgroup  $H = SU(4) \times SU(3) \times U(1)$  and contains 24 parameters. There are 24 scalars in the coset  $K$ . These scalars correspond to the non-compact generators  $Y_i$  for  $i = 1, \dots, 6, 9, \dots, 14, 17, \dots, 22, 25, \dots, 30$ . The subspace  $V$  consists of three generators which can be chosen to be  $Y_1, Y_{11}, Y_{21}$ . It is now easy to verify that a subgroup of  $H$  that commutes with these three generators is  $U(1) \times U(1) \times U(1)$ . So, the redundancy in the parametrization is given by  $H_0 = U(1)^3$ . We then find  $B = \frac{SU(4) \times SU(3) \times U(1)}{U(1) \times U(1) \times U(1)}$ . We can identify one of the  $U(1)$  in  $U(1)^3$  with the  $U(1)$  factor in  $H$ . Furthermore, we also choose to remove the remaining  $U(1)^2$  in  $H_0$  by moding out one  $U(1)$  factor from  $SU(4)$  and the other one from  $SU(3)$ . We are now left with  $B = \frac{SU(4)}{U(1)} \times \frac{SU(3)}{U(1)}$ . There are other ways of removing the redundancy other than that given here, but they are equivalent after redefining the scalars.

With all these, the coset representative for the coset  $K$  is given by

$$L = e^{a_1 c_3} e^{a_2 c_2} e^{a_3 c_3} e^{a_4 c_5} e^{\frac{1}{\sqrt{3}} a_5 c_8} e^{a_6 c_3} e^{a_7 c_3} e^{a_8 j_3} e^{a_9 j_2} e^{a_{10} j_3} e^{a_{11} j_5} e^{\frac{1}{\sqrt{3}} a_{12} j_8} e^{a_{13} j_{10}} \times \\ e^{a_{14} j_3} e^{a_{14} j_3} e^{a_{15} j_2} e^{a_{16} j_3} e^{a_{17} j_5} e^{\frac{1}{\sqrt{3}} a_{18} j_8} e^{a_{19} j_3} e^{a_{20} j_2} e^{a_{21} j_3} e^{b_1 Y_1} e^{b_2 Y_{11}} e^{b_3 Y_{21}}, \quad (153)$$

where the  $SU(4)$  generators  $j_i$ 's are defined in (97). The generators denoted by  $c_i$ 's generate the  $SU(3)$ . The explicit parametrization of both  $SU(4)$  and  $SU(3)$  can be found in [16].

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